Capital Structure and the Substitutability versus Complementarity Nature of Leases and Debt

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Abstract

At the most fundamental level, the capital structure irrelevance argument of Modigliani and Miller (1958) implies that the use of debt or leases should have no impact on firm values. However, the theoretical and empirical evidence is not conclusive. Thus, we re-examine the trade-off between leasing and debt and provide new insights into the conditions for their substitutability versus complementarity behaviors. We follow Grenadier (1996) and Leland and Toft (1996) to examine the interaction between firm capital structures and equilibrium contract pricing. Our model demonstrates that credit risk is instrumental for understanding the tradeoff between debt and leases.

Key words: leasing valuation, credit risk, endogenous default

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1 Introduction

One area of considerable interest in finance concerns the potential trade-off between leases and debt. At the most fundamental level, the capital structure irrelevance argument of Modigliani and Miller (1958) implies that the use of debt or leases should have no impact on firm values. In support of this position, numerous studies have presented theoretical models that assume the substitutability of debt and leases.[1] Empirically, these models have considerable support.[2]

While the substitutability of debt and leases is consistent with the capital structure irrelevance theory of Modigliani and Miller (1958), the theoretical and empirical evidence is not conclusive. For example, in an early empirical study, Ang and Peterson (1984) posit a leasing puzzle after finding a positive (complementary) relation between debt and leases. On the theoretical front, Lewis and Schallheim (1992) and Eisfeldt and Rampini (2009) develop models that imply that debt and leases may in fact be complementary. In Lewis and Schallheim (1992) complementarity arises from incentives inherent in the treatment of tax shields associated with debt and depreciation. In contrast, Eisfeldt and Rampini (2009) motivate their complementary view of debt and leases based on the heterogeneity of agency costs associated with firms with varying credit constraints. More recently, Schallheim, Wells, and Whitby (2013) empirically test the complementary versus substitutability of debt and leases using data on sale-and-leaseback transactions. Their analysis demonstrates that a substantial number of firms (approximately 42 percent of their sample) appear to exhibit a complementary relation between debt and leasing. As is evident from this brief synopsis of the literature, the debate over whether debt and leases are substitutes or complements remains unsettled. Thus, in this paper we re-examine the trade-off between leasing and debt usage to develop a theoretical model that provides new insights into the conditions that lead to the substitutability versus complementarity views of leases and debt.

The motivation for our model arises from the observation that a firm's capital structure

[1] For example, Myers, Dill, and Bautista (1976) provide theoretical justification for treating leases and debt as substitutes. Smith and Wakeman (1985) provide an informal list of characteristics of users and lessors that influence the leasing decision, and argue that the substitutability between debt and lease is affected by these characteristics. Please refer to Eisfeldt and Rampini (2009) for a more detail review.

[2] See for example, Bayliss and Diltz (1986), Marston and Harris (1988), Beattie, Goodacre, and Thomson (2000), Yan (2006), and more recently Agarwal et al. (2011).

decisions can have significant impact on its financial and operational contracts. As an example, consider the events on April 16, 2009 when General Growth Properties made history as one of the largest real estate Chapter 11 bankruptcy filings.[3] At the time, General Growth owned or managed over 200 shopping malls with balance sheet assets listed at over $29 billion. While creditors of General Growth were naturally concerned about the prospects of losses arising from the bankruptcy filing, tenants in General Growth malls that had secured leaseholds, which should have made them immune to problems associated with the lessor's bankruptcy, also expressed concern about the impact that the bankruptcy filing would have on their leasehold positions.[4] Similarly, Kulikowski (2012) notes that the near bankruptcy and eventual privatization of Quiznos in 2012 along with bankruptcy filings of other high profile franchise operators during the financial crisis raised awareness of franchisee exposure to capital structure decisions of their franchisors. Kulikowski (2012) quotes franchise lawyer Jeff Fabian as pointing out that "a franchisor's bankruptcy can significantly impact the success or failure of a franchisee's operations. From loss of supply of branded inventory, to loss of affiliation with the franchisor's trademark entirely, to loss of operational support, to customer confusion or defection as a result of less-than-flattering headlines, franchisor's bankruptcies can have real and long-term effects for the businesses of their franchisees."[5]

Using these examples as motivation, we provide novel insights into the debate concerning the substitutability or complementarity of debt and leases by developing a continuous-time structural model that endogenously considers, without ad hoc assumptions, the capital structure decisions (the choice of debt, equity, and leasing) of two firms that are linked through a financial contract. Our model generates new predictions about how a firm's capital structure can impact the terms of financial contracts and, in particular, provide unique insights into the conditions that would result from debt and leases being either complements or substitutes.

Our model is related to the growing recognition in the literature of the role that relationships among and between a firm's stakeholders can have on shaping a firm's financial decisions. For example, Titman (1984) and Maksimovic and Titman (1991) consider how a firm's capital structure can impact the types of contracts the firm has with its customers.

[3]See Hudson (2009)
[4]See Schaefers (2009).
[5]See Kulikowski (2012), pg. 1.

This stream of literature recognizes that when a firm has an interdependent relation with another firm (for example, a unique product that requires investments that might decline in value if the firm liquidates), then the firm may make capital structure decisions in order to maximize the value of these relations. Similarly, another line in the literature recognizes how firm capital structure decisions can impact management-labor relations. For example, Bronars and Deere (1991), Dasgupta and Sengupta (1993), and Hennessy and Livdan (2009) note that a firm's management can affect their bargaining power over labor unions by altering the firm's debt level to reduce the amount of surplus available to stakeholders. In addition, research using similar logic considers the role that capital structure decisions have on the firm's supply chain relationships (e.g. Kale and Shahrur (2007), Matsa (2010), and Chu (2012).)

More specifically, we propose a structural model based on the work developed by Leland and Toft (1996) and Agarwal et al. (2011) to effectively link both the landlord capital structure and tenant capital structure to the problem of determining the competitive lease rate. Similar to Leland and Toft (1996), we analyze the endogenous default problem by deriving the equilibrium lease rates given the default boundaries for both tenant and landlord.[6] In our discussion, we first determine the equilibrium lease rate (by equating the service flows of the leased asset to the lease payments) when the landlord can default but the tenant is risk-free. Next, we relate this rate to the equilibrium lease rate when both the landlord and tenant can default, which requires updating the landlord and tenant default boundaries. Identifying these boundaries leads to an in-depth discussion on the capital structure of each firm, which directly extends the analysis in Leland and Toft (1996) to include leases. As a result, our analysis provides direct insights into conditions that should prevail when leases and debt are observed as either substitutes or complements.

To preview our results, our model shows the role that potential landlord default plays in determining the competitive lease rate. Specifically, we identify how tenants are compensated (penalized) in the form of lower (higher) lease rates for increasingly (decreasingly) risky

[6]Our model significantly expands the model and analysis presented in Agarwal et al. (2011) by incorporating the non-trivial interactions of credit-risky tenants and landlords. To the best of our knowledge, this is the first attempt at recognizing the duel causality of capital structure decisions from both parties to a contract endogenously determining the contracting price.

financing decisions made by the landlord. We obtain a striking, yet consistent, contrast to previous studies in that debt and leases complement each other when the capital structures of both the landlord and the tenant are considered in the leasing problem. This finding is consistent with the conclusion obtained by Lewis and Schallheim (1992) in their one-period leasing model. Finally, our numerical implementation also facilitates an examination into the impact of changes in government tax policies upon lease rates. Specifically, we illustrate how differing tax environments can compensate (penalize) counterparties of the lease agreement through the lease rate.

Using property level and loan level information on mortgages contained in commercial mortgage backed securities, we empirically test several of the model's predictions. We make use of the ability to identify properties that are leased by single-tenants in order to isolate the impact of tenant capital structure on lease rates. By searching on tenant names, we identify publicly traded tenants with publicly available financial statements at the time of the lease. Thus, we are able to verify the prediction that lease rates are negatively related to tenant and landlord capital structures. We further document that lease maturity has a differential impact on lease rates based on the lessor's risk, as predicted by the model.

The remainder of the paper is organized as follows. Section 2 summarizes the existing literature on lease valuation. Section 3 presents the setting for determining lease rates and provides for the cases of a risky landlord, risk-less tenant (r_{RN}) and a risky landlord, risky tenant (r_{RR}) as well as a discussion on how they related. Section 4 describes the capital structure setup and Section 5 presents the endogenous decision rules to derive the optimal bankruptcy trigger levels for each firm. Section 6 presents a numerical implementation of the leasing model and discusses the comparative statics to assess the impact of relevant parameters on the term structure of lease rates. In Section 7, we present empirical evidence supporting the model's predictions and Section 8 concludes the discussion of the paper.

2 Literature Review

In this section, we survey the literature regarding the complex relationship between debt and leases. Additionally, we also discuss related literature about lease rate determination, and how a correct lease contract valuation model can be related to a firm's leasing policy.

As we mentioned earlier, the theory of corporate leasing policy dates back to Modigliani and Miller (1958). There, frictionless markets and no-arbitrage ruled out the importance of capital structure decisions on maximizing firm value. Of course, in realistic markets, frictions do exist and complicate the capital structure decisions for firms. In particular, a firm deciding between leasing and debt is presented with a number of differing tax incentives which confound the decision-making process.

Many works often regard debt and leases as interchangeable. Empirical studies supporting this conclusion include Bayliss and Diltz (1986), Marston and Harris (1988), Beattie, Goodacre, and Thomson (2000), and Yan (2006). In Bayliss and Diltz (1986), the authors conduct a survey of bank loan officers, presenting them with firms who use varying lease obligations and measure their willingness to make loans to these firms. Bayliss and Diltz (1986) quantitatively determine that \$1 of leases can substitute \$0.85 of debt. In Marston and Harris (1988), the authors examine the changes in debt and lease obligations and find that \$1 of leasing displaces approximately \$0.60 of non-leasing debt. More recently, Beattie, Goodacre, and Thomson (2000) use United Kingdom data to find that 1 British Pound of leasing displaces 0.23 British Pound of non-lease debt. In a more comprehensive study, Yan (2006) uses simultaneous-equation approach to examine the problem of debt or lease use. While utilizing a General Method of Moments (GMM) model to estimate parameters, Yan (2006) rejects the hypothesis that debt and leases are complements, but cannot reject the substitutability hypothesis. Additionally, Yan (2006) also finds that the degree of substitutability is greater for firms that pay no dividends (more asymmetric information), firms that have more investment opportunities (higher agency costs from underinvestment), and firms that have higher marginal tax rates (transferring tax shields is less valuable)

On the other hand, there are several studies which show how debt and leases can act as complements. Lewis and Schallheim (1992) propose a tax-based model that allows for low tax paying firms to sell excess tax shields to firms that place a much higher value on these tax deductions. By selling redundant tax shields, the lessee is motivated to increase its proportion of debt relative to an otherwise identical firm that does not use leasing. Eisfeldt and Rampini (2009) also find that debt and lease can be complements by focusing on the repossession advantage of leasing to those willing to lease to more financially constrained firms. However, the agency costs of leasing due to the separation of ownership and control of the leased assets counter-balances this effect. The net advantage accruing to lessors allows them to offer leases to more credit-constrained firms who will then choose to lease more of their capital than less constrained firms. As a result, debt and leases can be complements.

Additionally, several empirical studies find evidence showing how debt and leases can act as complements. In particular, debt and leases seem to be positively associated in the data. For example, Bowman (1980) observes a positive relationship between relative levels of debt and leases. Additionally, Ang and Peterson (1984) propose the so-called "leasing puzzle" and, in doing so, demonstrates a positive correlation between leasing and debt. Both of these findings demonstrate a complementary relationship between debt and leases.

As the above studies demonstrate, there appears to be no wide consensus regarding the precise relationship between leases and debt. One possible reason is that the lease contract is not correctly valued and the lease rate is not well determined. For example, traditional models of lease rates, beginning with Lewis and Schallheim (1992) and Grenadier (1996), have long recognized the importance of tenant default and hence tenant credit risk.[7] However, as noted above, leases are not one-sided contracts but rather specify rights and responsibilities of both the tenant and the landlord. For example, the typical commercial real estate lease specifies not only the amount of rent owed by the tenant but also the landlord's responsibilities in providing services associated with the contracted space. As a result, the typical lease creates the possibility that either party to the contract might default on the contract exposing both the landlord and the tenant to counterparty risk.

In a recent paper, Agarwal et al. (2011) focus on the tenant's default risk and its effect on the tenant's capital structure, assuming the landlord is default free. Their model is based on the framework originally proposed by Leland and Toft (1996) and examines the interaction of lessee financial decisions and lease rates. Our paper extends this framework to incorporate both lessor and lessee default risks into the term-structure of lease rates as well

[7]Recent work by Clapham and Gunnelin (2003), Ambrose and Yildirim (2008), and Agarwal et al. (2011) expanded on these models to explicitly incorporate the interaction of tenant credit risk and capital structure on the endogenous determination of lease rates. These models implicitly and explicitly recognize the risk that tenants may default on their lease obligations.

as its endogenous effect on both tenant and landlord capital structures. By this extension, our paper is also related to the works of correlated default modeling (e.g.: Zhou (2001), Yu (2007), Duffie and Singleton (1999), Das et al. (2007), Duffie et al. (2009)), and counterparty credit risk modeling (e.g.: Jarrow and Yu (2001)). However, none of these works consider lease rate term structure modeling and the implied joint capital structure decisions. In contrast, our paper demonstrates how capital structure decisions can endogenously impact other firms. In addition, our paper contributes to the research on correlated defaults in that the correlated default probability and lessor and tenant capital structures can be endogenously determined, while previous research is either based on reduced form models or exogenous structural models. The advantage of our model is its flexibility in capturing the credit risk interactions between landlord and tenant. Our model also can explain the phenomenon of credit contagion given the large amount of real estate leases utilized by firms (e.g.: Jorion and Zhang (2009)).

3 Determination of Lease Rates

We begin by defining a simple market environment for the purchase of space that fully captures the basic features of the commercial real estate leasing market. For ease of exposition, we describe the setting in terms of the traditional office leasing market but recognize that our model is easily generalizable to other property types (e.g. retail, industrial, etc.) as well as other assets that are commonly leased (e.g. commercial aircraft, computer equipment, etc.).

The office building owner (the landlord) holds the property in a firm financed with debt and equity. For the moment, we assume that the landlord and tenant capital structures are exogenously given in order to derive equations giving the equilibrium lease rates. Later, in Section 5, we relax this assumption by solving for the endogenous default boundary conditions that implicitly recognizes the tradeoff between debt and leases. We assume the property's future service flows are given by:

(1) $$\frac{dS_{BD}}{S_{BD}} = \mu_S dt + \sigma_S dW_S.$$

where S_{BD} represents the service flow before depreciation, μ_S is the drift rate of the service flow process, σ_S is the volatility of this process, and dW_S is the standard Brownian Motion under physical measure $\mathbb{P}$. On the other hand, if we reflect the economic (not accounting) depreciation of the leased asset in the drift rate of the service flow process (denoted by q), we can write the service-flow process after depreciation, S_{AD}, as:

$$\frac{dS_{AD}}{S_{AD}} = (\mu_S - q)dt + \sigma_S dW_S. \tag{2}$$

Without loss of generality, we assume that debt is in the form of a traditional mortgage secured by the property, and the mortgage is senior to any leasehold. Thus, the landlord is a credit-risk lessor. Additionally, we assume a default-free asset exists that pays a continuous interest rate r.

The landlord leases the property to a firm (the tenant). In the analysis below, we first define the equilibrium lease rate assuming a risk-free lessee (Case 1) and then extend the model to a credit-risk tenant (Case 2). Thus, our model highlights the complex interactions that result in contracts between credit-risky counterparties. In the following cases, we denote the periodic rent payment on a t-period lease contract between a credit-risk (R) lessor and a risk-free (N) lessee as r^t_{RN} and the rental payment on a lease originated between a credit-risk (R) lessor and a credit-risk (R) lessee as r^t_{RR}. Subscripts T, L which appear in notation to follow generally refer to tenant and landlord, respectively.

3.1 Case 1: The Risky Landlord and the Risk-Free Tenant

To derive a full equilibrium lease rate, we first model the case assuming a risk-free lessee and a credit-risk lessor. This scenario resembles the situation where a developer builds and leases an office building to the government. The lessor's credit risk results from his decisions regarding capital structure and thus a risk-free lessor is a special case where the lessor has no debt. Since the landlord's ability to provide the contracted service flow may be impacted by default, his default probability is considered in the formulation of the net cost of the lease contract.

Our motivation for considering landlord default is suggested by the General Growth bankruptcy example discussed above. As noted in the press reports concerning the General Growth bond default, tenants in properties where the owner faces financial stress are now discovering the risks associated with the default or bankruptcy of their landlord. For example, Sullivan III and Kimball (2009) point out that "if the lease was entered into after the landlord's mortgage (or, as is often the case, the lease provides that it is automatically subordinate to any mortgage), the lender's foreclosure action would automatically terminate the lease, wiping out the tenant's right to possession along with its investment in its leasehold improvements." As a result, when a landlord defaults on her mortgage, tenants may find that their leases are terminated. Sullivan III and Kimball (2009) also note that in recent years lenders often required standard subordination, nondisturbance and attornment (SNDA) agreements in leases as a condition of obtaining financing. These seemingly benign SNDA agreements often provide lenders (or purchases at foreclosure) significant rights with respect to the treatment of tenants and leases. For example, a standard lender initiated SNDA may limit the lender's liability in the event of foreclosure to complete lease contracted tenant improvements, or restrict or eliminate any purchase or renewal options specified in the lease.

In addition to risks associated with lessor default and foreclosure on debt, tenants also face the possibility that property owners may file for protection from creditors under the bankruptcy code. If a landlord files for reorganization under Chapter 11 of the bankruptcy code, then the tenant's lease contract is subject to Sections 365 and 363 of the Bankruptcy Code. These sections allow the bankruptcy trustee to either affirm or reject the lease. As a result, tenants with below market rents could find their leases terminated or property services suspended as part of an overall debt restructuring plan.[8]

To begin, we define distributions with respect to the first passage time u to the landlord's default boundary $V_{L,B}$ from the landlord's present un-leveraged firm value V_L. Accordingly, we let $p_L(u; V_L, V_{L,B})$ denote the landlord's cumulative survival probability and define

[8]According to Title 11, Chapter 3, Subchapter IV, Section 365 (h)(1), tenants in a lease rejected by the trustee may retain their rights to occupy the space as defined by the lease, but the landlord is released from providing services required under the lease. The tenant will then have to contract separately for those services and may offset the costs of those services from future rent payments. (See Anderson (2014) and Eisenbach (2006).) However, Eisenbach (2006) further notes that tenants in a sublease do not have protection under Section 365(h)(1) and thus would have no rights to continue occupying the space if the trustee rejects the original lease. Harvey (1966) also discusses the rights of tenants upon landlord breach under California law.

$f_L(u; V_L, V_{L,B})$ to be the probability density function of landlord default. We express the time of landlord default as τ_L. The landlord's expected net cost of providing the lease from time 0 until maturity t is the expected present value of the service flows before depreciation minus the tax-shield benefit associated with the depreciation expense before default plus the expected cost claimed by the tenant if the landlord defaults:

$$
\begin{aligned}
&\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru}[S_{BD}(u)\mathrm{d}u - \chi_L Tax_L(S_{BD}(u)\mathrm{d}u - S_{AD}(u))]\mathbb{1}_{\{\tau_L > u\}}\mathrm{d}u\right] \\
&+\widetilde{\mathbb{E}}\left[\rho_L^t\left(\int_{\tau_L}^t e^{-ru}[S_{BD}(u) - \chi_L Tax_L(S_{BD}(u) - S_{AD}(u))]\mathrm{d}u\right)\mathbb{1}_{\{\tau_L \leq t\}}\right] \\
&= \int_0^t e^{-ru}\left[S_{BD}(u)du - \chi_L Tax_L\left(S_{BD}(u)du - S_{AD}(u)\right)\right]p_L(u; V_L, V_{L,B})\,\mathrm{d}u \\
&+ \int_0^t e^{-ru}\mathrm{Damage}_u f_L(u; V_L, V_{L,B})\,\mathrm{d}u,
\end{aligned}
\tag{3}
$$

where $\int_0^t S_{BD}(u)e^{-ru}du$ represents the present value of the service flows before depreciation from time 0 to time t discounted by the risk-free rate under risk-neutral measure $\widetilde{\mathbb{P}}$, and $\int_0^t S_{AD}(u)e^{-ru}du$ represents the present value of the service flow after depreciation under the risk-neutral measure $\widetilde{\mathbb{P}}$. $\widetilde{\mathbb{E}}(\cdot)$ is the expectation under $\widetilde{\mathbb{P}}$. The difference between these two terms is the depreciation cost of the leased asset from time 0 to t. Tax_L is the corporate tax rate for landlord and χ_L is the depreciation adjustment factor that reconciles the government mandated accounting depreciation to the actual physical depreciation.[9] The indicator function

$$
\mathbb{1}_{\{\tau_L \leq t\}} = \begin{cases} 1, & \text{if } \tau_L \leq t, \\ 0, & \text{otherwise.} \end{cases}
$$

highlights when a default occurs. ρ_L^t denotes the recovery rate of the lease contract upon landlord default which may be a function of the maturity t. Additionally, Damage_u refers to the landlord's cost upon default at time u.

The first term in equation (3) is the the expected present value of the service flows before

[9]See Agarwal et al. (2011).

depreciation minus the tax-shield benefit associated with the depreciation expense before default, and the second term is the expected cost claimed by tenants upon landlord default.

If the landlord or debtor rejects the lease, we have to consider two scenarios: First, the tenant leaves the leased property and files a claim equal to her loss due to landlord's default. That loss might be the due to the inability to use the leased property or the loss associated with having to temporarily stop its business operation. In the second scenario, the tenant remains in the leased property and the landlord continues to pay the cost of providing the contractual service flow. However, the tenant may be responsible for additional costs, such as power, heat and trash disposal and thus we can assume that damage is a proportion of the present value of future service flows.

We define Damage_u in (3) to be a percentage of the present value of future service flows from default time u to t

$$\text{Damage}_u = \rho_L^t \int_u^t e^{-rv} \left[S_{BD}(v) - \chi_L Tax_L \left(S_{BD}(v) - S_{AD}(v) \right) \right] \mathrm{d}v \tag{4}$$

For simplicity, we assume the recovery rate ρ_L^t is constant and independent of t, i.e., $\rho_L^t = \rho_L$. Since the leased property is still in the hands of the landlord, he is responsible for the depreciation expense. Thus, the sum of the two terms in equation (3) is the expected net cost of providing the leased property from the landlord's perspective, recognizing the tax-shield benefit associated with the depreciation expense.

In a competitive market, the expected net cost of the lease exactly equals the present value of the future lease payments if the tenant does not default. Thus the expected cost of the lease is $\int_0^t r_{RN}^t e^{-ru} du = r_{RN}^t (\frac{1-e^{-rt}}{r})$, where r_{RN}^t denotes the operating lease rent with maturity t for a combination of a risky landlord and a risk-free tenant. We can solve for the lease rate r_{RN}^t by setting equation (3) equal to $r_{RN}^t (\frac{1-e^{-rt}}{r})$. Thus, assuming the asset service flow follows (2), then the lease rate is:[10]

$$
\begin{aligned}
r_{RN}^{t} \quad = \quad & \frac{r}{1-e^{-rt}} \\
& \times\Bigg[(1-\chi_L Tax_L)\int_0^t S_{BD}(0)e^{(\mu_S-r-\delta\sigma_S)u}(1-F(u;V_L,V_{L,B}))\mathrm{d}u \\
& +\chi_L Tax_L\int_0^t S_{AD}(0)e^{(\mu_S-r-q-\delta\sigma_S)u}(1-F(u;V_L,V_{L,B}))\mathrm{d}u \\
& +\rho_L^t\Bigg((1-\chi_L Tax_{LT}) \\
& \times\frac{S_{BD}(0)}{\mu_S-r-\delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-\delta\sigma_S)u}\ f_L(u;V_L,V_{L,B})\mathrm{d}u\right) \\
& +\chi_L Tax_L\frac{S_{AD}(0)}{\mu_S-r-q-\delta\sigma_S} \\
\text{(5)} \qquad & \times\left(e^{(\mu_S-r-q-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-q-\delta\sigma_S)u}f_L(u;V_L,V_{L,B})\mathrm{d}u\right)\Bigg)\Bigg].
\end{aligned}
$$

where δ denotes the market price of risk for the service value process. $F(u;V_L,V_{L,B})$ is landlord's cumulative default probability, and $f_L(u;V_L,V_{L,B})$ is landlord's default probability.

3.2 Case 2: The Risky Landlord and The Risky Tenant

We now examine the lease contract assuming a credit-risk tenant. Recall that the present value of a lease with maturity t to a risk-free tenant is $r_{RN}^t(\frac{1-e^{-rt}}{r})$. When both landlord and tenant have credit risk, we must calculate the present value of the lease rate r_{RR}^t from origination to the tenant default time u, and the recovery of the remaining lease rentals from time u to maturity time t. Under these conditions, we can express the value of the default-risky lease as:

$$
\text{(6)} \qquad \int_0^t e^{-ru}r_{RR}^t\left(1-F_T(u;V_T,V_{T,B})\right)\mathrm{d}u+\int_0^t e^{-r\tau}\rho_R^t R_{T,RR}^{t-u}f_T(u;V_T,V_{T,B})\mathrm{d}u
$$

where $F_T(u;V_T,V_{T,B})$ is the tenant's cumulative default probability up to time u under measure $\widetilde{\mathbb{P}}$, $f_T(u;V_T,V_{T,B})$ is the tenant's instantaneous default probability under measure $\widetilde{\mathbb{P}}$

[10]See the Appendix for the proof.

at time u, and ρ_R^t is the tenant's recovery rate. $R_{T,RR}^{t-u}$ is the present value of the remaining lease payments, and it can be expressed as $r_{RR}^t(\frac{1-e^{-r(t-u)}}{r})$. The first term in (6) represents the expected discounted lease payment flows from 0 to u. The second term represents the expected discounted value of the remaining lease payments after default.

Following the arguments in Grenadier (1996) that any two methods of selling an asset's service flow for t-years must have the same value, then in equilibrium the lease values in case 1 must equal case 2. Thus, we can combine equations (5) and (6)

$$\begin{aligned} & r_{RN}^t\left(\frac{1-e^{-rt}}{r}\right) \\ = \; & \int_0^t e^{-ru} r_{RR}^t \left(1 - F_T(u; V_T, V_{T,B})\right) \mathrm{d}u + \int_0^t e^{-ru} \rho_R^t R_{T,RR}^{t-u} f_T(u; V_T, V_{T,B}) \mathrm{d}u, \end{aligned}$$

and express the relationship between the lease rates in cases 1 and 2 with a maturity of t as:

$$r_{RR}^t = r_{RN}^t \left[\frac{1-e^{-rt}}{(1-e^{-rt}) - (1-\rho_R^t)\left(G_T(t) - F_T(t)\, e^{-rt}\right)}\right], \tag{7}$$

where

$$G_T(t) := \int_0^t e^{-ru} f_T(u; V_T, V_{T,B}) \mathrm{d}u. \tag{8}$$

Equation (7) shows the relation between the risky lease rate and the risk-free lease rate. The denominator represents the discount factor associated with a default-risky lease, and the numerator is the discount factor associated with a risk-free lease. The first part of denominator is the default-free discount factor which is the same as the numerator. The second part is the loss rate $(1-\rho_R^t)$ times a difference of discounted default probabilities; a positive quantity [11]. From this equation, when the lessee's default probability increases, implying $(G_T(t) - F_T(t)\, e^{-rt})$ increases, the value of the denominator decreases. Hence, the

[11] Note that

$$\left(G_T(t) - F_T(t)\, e^{-rt}\right) = \int_0^t e^{-ru} f_T(u; V, V_B) \mathrm{d}u - \int_0^t e^{-rt} f_T(u; V, V_B) \mathrm{d}u = \int_0^t \left(e^{-ru} - e^{-rt}\right) f_T(u; V, V_B) \mathrm{d}u > 0.$$

risky lease rate increases to compensate for the increase in default probability. In addition, when the expected recovery rate increases, the lessor recovers more when the lessee defaults, and thus, the risky lease rate decreases, all else being equal.

4 Capital Structure

By extending Agarwal et al. (2011), the purpose of our analysis is to incorporate the effects of lease credit risk on both the landlord and tenant capital structure in order to determine its net effects on the equilibrium term structure of lease rates. [12] The underlying framework for our landlord and tenant capital structures is the continuous time structural model of Leland and Toft (1996). In this section and in Section 5, we adapt the Leland and Toft (1996) approach for both the landlord and the tenant to accommodate the lease agreement between both parties.

Following Merton (1974), Black and Cox (1976), Brennan and Schwartz (1978) and Leland and Toft (1996), we assume the landlord has productive assets, one of which is the leased asset delivering service flows described in (1). The un-leveraged value of the landlord's firm (V_L) follows a continuous diffusion process with constant proportional volatility σ_{V_L}:

$$\frac{\mathrm{d}V_L}{V_L} = \left(\mu_{V_L}(t) - \delta_{V_L}\right)\mathrm{d}t + \sigma_{V_L}\mathrm{d}W_{V_L}, \tag{9}$$

where $\mu_{V_L}(t)$ denotes the landlord's total expected rate of return on asset V_L, δ_{V_L} is the landlord's constant fraction of value paid out to all security holders, and $\mathrm{d}W_{V_L}$ is the increment of a standard Brownian motion under the physical measure $\mathbb{P}$.[13]

We assume the landlord's capital structure is composed of debt and equity. Consider a single debt issue with maturity t, having periodic coupon ($c_L(t)$) and principal ($p_L(t)$) payments. Upon bankruptcy, the bondholder forecloses on the debt and recovers a fraction $\rho_{L,D}(t)$ of the firm's net asset value of $\widetilde{V}_{L,B}$, where $\widetilde{V}_{L,B}$ equals the net asset value after bankruptcy costs plus the present value of lessor's recovery lease payments at the time of

[12] For the Tenant's capital structure incorporating the lease and debt, refer to Section IV of Agarwal et al. (2011).

[13] Placing the leased asset inside a firm mirrors the market practice of securitizing real estate assets in a REIT structure.

default. In other words, $\rho_{L,D}(t)$ is the bondholder's recovery rate for a debt with maturity t. Thus, we can write the value of risky debt as:

$$\begin{aligned} d_L(V_L;V_{L,B},t) &= \int_0^t e^{-ru}c_L(t)\left(1-F_L(u;V_L,V_{L,B})\right)\mathrm{d}u + p_L(t)e^{-rt}\left(1-F_L(t;V_L,V_{L,B})\right) \\ &\quad + \int_0^t e^{-ru}\rho_{L,D}(t)\widetilde{V}_{L,B}f_L(u;V_L,V_{L,B})\mathrm{d}u \end{aligned} \tag{10}$$

If the firm does not declare bankruptcy, then the first term on the right hand side of (10) represents the present value of coupon payments, and the second term represents the present value of the principal payment, respectively. The third term represents the present value of the net asset value accruing to the debt holders if bankruptcy occurs. Thus, we can rewrite equation (10) as:

$$\begin{aligned} d_L(V_L;V_{L,B},t) &= \frac{c_L(t)}{r}\left(1-e^{-rt}\right) - \frac{c_L(t)}{r}\left(G_L(t)-F_L(t)e^{-rt}\right) + e^{-rt}p_L(t)\left(1-F_L(t)\right) \\ &\quad + \int_0^t e^{-ru}\rho_{L,D}(t)\widetilde{V}_{L,B}f_L(u;V_L,V_{L,B})\mathrm{d}u. \end{aligned} \tag{11}$$

We assume that when landlord defaults, he receives an automatic liquidation stay from the bankruptcy court. Given this assumption, we have:

$$\widetilde{V}_{L,B} = \left(1-\alpha_L\right)V_{L,B} \tag{12}$$

where α_L is the proportion of firm value loss when landlord firm goes bankrupt, and (12) is consistent with the ordinary trade-off theory of optimal capital structure theory.

As with the landlord, we assume the tenant firm has productive assets whose un-leveraged value V_T follows a continuous diffusion process

$$\frac{\mathrm{d}V_T}{V_T} = \left(\mu_{V_T}(t) - \delta_{V_T}\right)\mathrm{d}t + \sigma_{V_T}\mathrm{d}W_{V_T}. \tag{13}$$

We assume that the tenant's capital structure consists of leases, debt, and equity.[14] Suppose

[14] In an operating lease, the present value of lease expenses are not listed on the debt side of the balance sheet and the operating lease expenses for the future 5 years are only listed as a footnote of the balance sheet. However, in terms of cash flows, the lessee firm will expend lease payments in exchange for the leased asset's service flows that generate operating cash flows for the firm. Therefore, in terms of cash flows, we

the tenant firm writes an operating lease contract maturing at time t for an additional asset. The lease contract value $l(V_T; V_{T,B}, t)$ is equal to expression (6), i.e.,

(14)
$$l(V_T; V_{T,B}, t) = \int_0^t e^{-ru} r_{RR}^t \left(1 - F_T(u; V_T, V_{T,B})\right) \mathrm{d}u + \int_0^t e^{-r\tau} \rho_R^t R_{T,RR}^{t-u} f_T(u; V_T, V_{T,B}) \mathrm{d}u.$$

For describing the tenant's debt, we follow the notation used for the landlord above. More specifically, for a single debt issue with maturity t, having periodic coupon ($c_T(t)$) and principal ($p_T(t)$) payments, we can write the value of the tenant's risk debt as:

$$\begin{aligned} d_T(V_T; V_{T,B}, t) &= \int_0^t e^{-ru} c_T(t) \left(1 - F_T(u; V_T, V_{T,B})\right) \mathrm{d}u + p_T(t) e^{-rt} \left(1 - F_T(t; V_T, V_{T,B})\right) \\ &+ \int_0^t e^{-ru} \rho_{T,D}(t) \widetilde{V}_{T,B} f_T(u; V_T, V_{T,B}) \mathrm{d}u, \end{aligned} \tag{15}$$

with analogous definitions (to the landlord case) for all quantities in the expression above. Note that Leland and Toft (1996) and Agarwal et al. (2011) follow similar notation to describe a firm's capital structure.

In the next section, we illustrate how the Leland and Toft (1996) endogenous default boundaries for both the landlord and tenant can be determined within our setting.

5 Determining the Endogenous Default Boundaries

As in Leland and Toft (1996), we adopt a stationary debt structure for both tenant and landlord. As such, we consider an environment where each firm continuously sells a constant amount of new debt with maturity of T years from issuance, which it will redeem at par upon maturity (if no default has occurred). In the following, we adopt the notation from Leland and Toft (1996) to describe the stationary debt structure. Let $T_{i,D},\ i = T, L$ denote the debt maturity for the tenant and landlord respectively. More specifically, we let $P_i,\ i = T, L$ denote the total principal amount of outstanding debt for tenant and landlord, respectively and let $p_i = P_i / T_{i,D}$ denote the amount of new debt issued per year. Similarly, the total coupon payment for tenant and landlord respectively is $C_i,\ i = T, L$ per year with constant

treat the present value lease expenses as a part of the lessee firm's debt side on the balance sheet.

coupon $c_i = C_i/T_{i,D}$. We begin by describing the tenant's endogenous default boundary followed by the landlord's endogenous default boundary.

For the tenant, the net asset value upon bankruptcy takes the form

$$\widetilde{V}_{T,B} = (1-\alpha_T)V_{T,B} - \rho_R \Omega_R \left(\frac{1-e^{-r(T_L-\tau_T)}}{r} \right),$$

where τ_T is the tenant default time, ρ_R is the recovery rate for lease payments (to the landlord) upon tenant default and Ω_R is the total lease payment per year and $r_{RR}^{T_L} = \Omega_R/T_L$ is the constant lease rate[15]. The first term is the asset value after bankruptcy costs and the second term represents the cash flow recovered by the landlord when the tenant defaults.

Following the derivation of equation (17) in Agarwal et al. (2011), we note that the tenant's endogenous bankruptcy boundary appears as

$$V_{T,B}^* = \frac{\frac{\Omega_R}{r}(K_1^{T_L} - K_2^{T_L}) - K_3 - K_4 + M - \left(P_T - \frac{C_T}{r}\right) K_1^{T_{T,D}} - \left(\frac{C_T}{r}\right) K_2^{T_{T,D}}}{1 + \alpha_T x_T - (1-\alpha_T)K_2^{T_{T,D}}}, \tag{16}$$

with the distinguishing feature that, within our current analysis, the total lease payments per year Ω_R is dependent upon the landlord's optimal bankruptcy boundary $V_{L,B}$ which is not present in the corresponding lease rate in Agarwal et al. (2011).[16] Indeed, the lease rate $r_{RR}^{T_L}$ found using equations (5) and (7) is a function of the landlord's bankruptcy boundary $V_{L,B}$. Thus, in this section, we identify the endogenous, optimal bankruptcy boundary $V_{L,B}^*$ (and hence the endogenous capital structure) for the landlord that is inserted into equations (5) and (7) in order to determine the lease rate $r_{RR}^{T_L}$.

Similar to the tenant, we assume the landlord trades off the tax benefits and the bankruptcy costs of debt financing. Since we incorporate lease financing into the capital structure decision, the tax deductibility benefit of the landlord equals the interest expense on the debt and the depreciation expense of the leased asset. Following Leland (1994), the total firm

[15]The reasoning here for leases is the same as in Leland and Toft (1996) for debt. Letting $l(V_L; V_{L,B}, t)$ denote the lease contract value with maturity t, the value of all outstanding leases (under our stationary debt and lease structure) is $\int_{t=0}^{T} l(V_L; V_{L,B}, t)\mathrm{d}t$. If one year passes, the total lease payment owed is approximately the Riemann sum $r_{RR}^{T_L}\triangle t + \ldots + r_{RR}^{T_L}\triangle t$ of $(T_L/\triangle t)$ terms. The exact payment is the limit of the Riemann sum equal to $r_{RR}^{T_L} T_L$. Now, set the continuous lease rate as $r_{RR}^{T_L} := \Omega_R/T_L$. Thus, Ω_R represents the total lease payment per year.

[16]The definitions of K_1^T, K_2^T, K_3, K_4, x_T, and M in equation (16) appear in the Appendix.

value of the landlord ($v_L(V_L; V_{L,B})$) equals the un-leveraged firm value plus the tax benefit of debt and lease financing minus the bankruptcy cost during the observation period:

$$\begin{aligned} v_L(V_L; V_{L,B}) = V_L + Tax_L \left(\frac{C_L}{r} + \frac{\text{Dep}}{r}\right)\left(1 - \left(\frac{V_{L,B}}{V_L}\right)^{x_L}\right) \\ - (\alpha_L V_{L,B} + \text{damage by default})\left(\frac{V_{L,B}}{V_L}\right)^{x_L} \end{aligned} \tag{17}$$

where V_L is the un-leveraged firm value, Dep is the depreciation, and x_L is defined in the Appendix. The second term in (17) represents the tax-benefits associated with interest rate expense and depreciation expense given that the landlord does not default. The third term in (17) is the bankruptcy cost given that the landlord defaults and includes bankruptcy costs documented by Warner (1977) and the damage compensation[17] to the tenant for the landlord's default. To model the periodic depreciation expense Dep we assume the leased asset is linearly depreciated and the landlord has a stationary lease structure; thus, the total depreciation expense for the life of the leased asset (T_{Life}) is $\widetilde{\mathbb{E}}\left[\int_0^{T_{\text{Life}}} S_{BD}(u)e^{-ru}du - \int_0^{T_{\text{Life}}} S_{AD}(u)e^{-ru}du\right]$. If we amortize the total expense to a periodic expense, the periodic expense Dep is $\widetilde{\mathbb{E}}\left[r \times \left(\int_0^{T_{\text{Life}}} S_{BD}(u)e^{-ru}du - \int_0^{T_{\text{Life}}} S_{AD}(u)e^{-ru}du\right)\right]$ because Dep$/r$ is the total life-long depreciation expense. In this setting, equation (17) is consistent with traditional capital structure trade-off theory that assumes the tax-shield benefit has a positive effect on firm value while bankruptcy costs have a negative effect.

We apply the smoothing-pasting condition in Leland and Toft (1996) and solve for the endogenous default boundary, $V_{L,B}$. Let[18]

$$\left.\frac{\partial E_L\left(V_L; V_{L,B}, T_{L,D}\right)}{\partial V_L}\right|_{V_L = V_{L,B}} = 0 \tag{18}$$

By solving equation (18), we find the endogenous bankruptcy boundary as:

$$V^*_{L,B} = \frac{(C_L/r)(A/(rT_{L,D}) - B) - AP_L/(rT_{L,D}) - (Tax_L(C_L + \text{Dep})/r + \text{damage by default})x_L}{1 + \alpha_L x_L - (1 - \alpha_L)B}, \tag{19}$$

[17]See the Appendix for a discussion about how this term is calculated for the numerical implementation.

[18]Letting $E_i, D_i,\ i = T, L$ denote tenant and landlord aggregate equity and debt value respectively, we have $v_L = E_L + D_L$, therefore, $E_L = v_L - D_L$

where A and B are defined in the Appendix and coincide with the same identifications established in Leland and Toft (1996). We then simultaneously solve for the landlord and tenant optimal bankruptcy levels (and resulting capital structures) by equations (19) and (16).

6 Numerical Implementation

In this section, we discuss a numerical implementation of our model. The construction of our model facilitates a separation of the interdependency between the landlords's and tenant's capital structure in determining the competitive lease rate. We divide the numerical implementation into two parts: First, we find the optimal endogenous bankruptcy boundary for the landlord $V^*_{L,B}$. Second, we use $V^*_{L,B}$ to calculate, respectively: (a) the risky landlord and risk-less tenant lease rate r_{RN} via equation (5); (b) the risky landlord and risky tenant lease rate r_{RR} via equation (7); and (c) the tenant's optimal endogenous boundary $V^*_{T,B}$ via equation (16). We then use the optimal boundaries $V^*_{L,B}$ and $V^*_{T,B}$ to calculate the tenant and landlord debt and equity values.

Our numerical implementation to determine $V^*_{L,B}$ is similar to the procedure carried out in Leland and Toft (1996). We begin by solving for the landlord's endogenous default boundary values ($V_{L,B}$) for a set of debt contracts characterized by the combination of principal and coupon (P_L, C_L) taken over the principal range $[0.5, 100]$ with steps $\triangle P_L = 0.5$. As in Leland and Toft (1996), we assume the coupon (C_L) is set so that newly-issued debt sells at par value $(d_L(V; c_L, p)|_{V_L=V_L(0)} = p_L$, where $p_L = P_L/T_{L,D}$ and $c_L = C_L/T_{L,D}$.) We use the bisection method to solve $d_L = p_L$ in order to obtain C_L for a given P_L. After obtaining the debt contract pair (P_L, C_L), we then calculate the corresponding endogenous default boundary $V_{L,B}$. Once we obtain the set of endogenous default boundaries that correspond to the set of debt principal and coupon contracts, we then select the debt contract (P_L, C_L) that maximizes the landlord's value $v_L(V_L; V_{L,B})$.

Given the landlord's endogenous boundary $V^*_{L,B}$, we then calculate the equilibrium lease rate and find the tenant's optimal capital structure. The numerical method for doing so, follows the procedure carried out in Agarwal et al. (2011). Recall that Leland and Toft (1996) demonstrate that a firm's optimal default boundary V_B can be calculated given the debt contract combination (P, C). However, from (16), we see that the tenant's optimal default boundary $V^*_{T,B}$ also depends upon the risky lease rent $r^{T_L}_{RR}$ as well as the debt contract. Thus, even if we fix P_T, we cannot directly solve for C_T satisfying $d_T(V; V_{T,B}, t) = p_T$ since $r^{T_L}_{RR}$ is also unknown.[19] It is useful to observe that equation (7) is equivalent to the requirement that newly issued leases are issued at their "par" value, i.e., equal to the expected present value of service flows. Thus, (7) is the natural extension for leases to the condition in Leland and Toft (1996) that requires new debt to be issued at "par" value. As a result, we input into $d_T(V; V_{T,B}, t) = p_T$ the value $r^{T_L}_{RR}$ that satisfies (7). In other words, the numerical task is to find the combination of c_T and $r^{T_L}_{RR}$ such that both debt and leases equal their par value for a given P_T.

Our extension of Leland and Toft (1996) involves solving a two-dimensional system of nonlinear equations as follows. First, we specify a principal and coupon range: $[0.5, 100] \times [(0.01)P, (0.1)P]$. We then fix the pair (P, C) and numerically solve (via the bisection method) equation (7) for $r^{T_L}_{RR}$.[20] Upon obtaining a solution to (7), we then check whether the value $r^{T_L}_{RR}$ also satisfies $d_T(V; V_{T,B}, t) = p_T$. If it does, then the pair $C_T, r^{T_L}_{RR}$ represents a solution to the two-dimensional system. If $r^{T_L}_{RR}$ does not satisfy $d_T(V; V_{T,B}, t) = p_T$ for the fixed set (P_T, C_T), we record this error, increment the coupon by $\triangle C$ and repeat the process.[21] We continue this process until $d_T(V; V_{T,B}, t) = p_T$ is satisfied or until $C_T = (0.1)P_T$.[22]

After obtaining P_T, C_T, and $r^{T_L}_{RR}$, we then calculate the endogenous tenant default boundary $V_{T,B}$ and capital structure corresponding to the pair (P_T, C_T) that maximizes the firm value $v_T(V_T; V_{T,B})$. The endogenous boundary corresponding to this capital structure is the optimal endogenous boundary for the tenant $V^*_{T,B}$. With this boundary, we then calculate the value of the tenant's debt and equity.

Table 1 presents the base case parameters used in the analysis to follow. Our base case

[19] The "correct" $r^{T_L}_{RR}$ satisfies (7).

[20] We note that the functions G_T and F_T are also functions of $r^{T_L}_{RR}$ through $V_{T,B}$. This significantly complicates the equation.

[21] We set $\triangle C = 0.01$.

[22] If $C = (0.1)P$ and we have not found a solution, we consider the pair $(C, r^{T_L}_{RR})$ corresponding to the smallest recorded error to be the approximate solution to the two-dimensional system.

parameters match those in the literature allowing for comparison of our results with previous studies [23]. Table 2 shows the relationship between the probability of default on the lessor's existing debt and the lease term structure. Specifically, we consider three cases of tenant debt: short-term ($T_{T,D} = 5$ years), medium-term ($T_{T,D} = 10$ years) and long-term ($T_{T,D} = 20$ years) across short- and medium-term lease maturities ($T_L = 5, 10$), assuming the landlord's debt maturity remains fixed at 5-years. Later, we relax this assumption and consider the effect of the landlord moving from short-term debt (5-years) to medium-term debt (10-years). In Table 2, the third row within each tenant debt block displays the optimal endogenous default boundaries for the landlord and tenant. The other rows consider alternative exogenous landlord default boundary values and the corresponding implied endogenous tenant default boundary, default probability, and lease rate. As will be noted below, the interactive effects of tenant and landlord default probabilities with lease rates are non-linear and depend upon the lease term (5-years or 10-years).

6.1 Impact of Landlord Default Probability.

The first column in Table 2 shows the landlord's bankruptcy boundary with the third row in each block being the endogenous default boundary. As expected, the landlord's default probability (column 2) increases as the default boundary increases. Columns (3) and (4) show the tenant's implied endogenous default boundary and probability that correspond to the landlord's default boundary while columns (5) and (6) show the equilibrium lease rates that correspond to a risk-free tenant ($r_{RN}^{T_L}$) and a tenant with credit-risk ($r_{RR}^{T_L}$), respectively.

As expected, we see that as the landlord's default probability increases, the equilibrium lease rate declines regardless of lease maturity. The effect of a shift in landlord risk is most evident under the case where the tenant is risk-free and the lease is long-term (10-years). In this scenario, the tenant has no default risk and thus the tenant's capital structure has no impact on the equilibrium lease rate ($r_{RN}^{T_L}$). As a result, an increase in landlord default probability from 0.1% to 36% results in a 19.35% decrease in the lease rate (from 0.638 to 0.515). However, as expected, shorter term leases mitigate the impact of landlord credit risk and thus the impact of an increase in counterparty risk is lower. For example, when the lease

[23] See for example Agarwal et al. (2011).

maturity is only 5-years, the lease rate declines only 8.2% as landlord default probability increases. A similar, but less dramatic effect occurs when the tenant is not risk-free ($r_{RR}^{T_L}$). However, the effect is complicated since now the tenant's capital structure also impacts the lease rate. Thus, as intuition suggests we conclude that tenants face lower equilibrium lease rates as their counter-party's risk increases and this risk increases with exposure to the landlord through lease maturity. Furthermore, these results confirm that credit contagion can be amplified through long-term off balance sheet contracts.

Finally, columns (7) through (10) show the endogenous tenant capital structure that results from contracting with a risky landlord. Overall, we observe that the use of leverage increases as the lease counter-party risk increases. For example, when both lease and tenant debt are long-term, the tenant's leverage ratio increases 74.58% (from 0.1503 to 0.2624) in response to an increase in the landlord's default probability. This phenomenon conforms with intuition that tenants have an increasing preference for debt as landlord riskiness increases.

6.2 Impact of Tenant and Landlord Debt Maturity.

Table 2 also provides insights into the impact on lease rates to changes to tenant debt maturity. To do so, we focus on the optimal endogenous landlord default boundary case (row three in each tenant debt maturity block highlighted in italics). First and intuitively obvious, Table 2 shows that tenant default probabilities increase with longer debt maturities (rising from 14.9% to 66.2% as maturity increases from 5-years to 20-years). Next, we consider the increase in tenant debt maturity from 5-years to 20-years and note that the tenant's capital structure also impacts the competitive lease rate. Comparing lease rates for short-term debt and long-term debt evaluated at the landlord's optimal endogenous default boundary (and holding all else constant), we see that lease rates are positively related to tenant debt maturity irrespective of the lease maturity date and the landlord default boundary. In other words, the results show that landlords are compensated in the form of higher lease rates for riskier tenant firms; this intuitive phenomenon was also observed in the risk-less landlord case examined by Agarwal et al. (2011).

Just as the tenant's capital structure impacts the competitive lease rate, financing decisions made by the landlord also influence lease rates. To illustrate these effects, Table 3 compares the landlord and tenant default probabilities and lease rates assuming the landlord issues short or medium term debt (5-years and 10-years) while the tenant issues short, medium, and long-term debt (5, 10, and 20-years). The results clearly demonstrate that, as predicted, long-term debt issuance by the landlord, which effectively increases the landlord's credit risk, reduces the lease rate, i.e., the tenant lease payment is reduced for riskier landlord firms. For example, when the lease is short term (5-years) and the landlord and tenant use short-term debt, the endogenous landlord default boundary is 43.84 with an implied default probability of 11.6%. However, as the landlord's debt maturity increases to 10-years the endogenous landlord default boundary increases to 47.17 with an implied default probability of 38.7%. This increase of landlord default risk translates into a lower lease rate (0.787 versus 0.781). However, we observe an interesting non-linear phenomenon on lease rates as tenant debt maturity changes. For example, holding landlord debt maturity constant at 10-years, the 5-year maturity lease rate first declines (from 0.781 to 0.779) as tenant debt maturity increases from 5 to 10-years and then rises (from 0.779 to 0.789) as debt maturity increases to 20-years. As a result, Table 3 reveals interesting new insights regarding the term structure of lease rates that have been ignored in previous studies that did not consider the endogenous counter-party risks.

6.3 Debt and Lease as Complements and Substitutes

Table 4 presents the main results of debt and leases as complements and substitutes. In Panel A, we show the interaction between leasing and the default risky firm's (both landlord and tenant) choice of optimal capital structure for different debt and lease maturities. In the case of a low default risk tenant, we document the complementary nature of debt and leases. For example, with the tenant debt maturity fixed (say, $T_{T,D} = 5$), then an increase in the lease maturity from 5 years to 10 years decreases the lease rate from 0.7871 to 0.6621, increases in tenant's default probability from 0.149 to 0.173, increases lease value from 3.2076 to 4.2288, and finally increases the debt value from 47.4267 to 52.7914. The positive relationship between lease and debt values (i.e. $\Delta L/\Delta D = 18.99\%$) posits the complementary effect. However, when the default probability of the tenant is increased by increasing the debt maturity of the tenant to $T_{T,D} = 10$ and $T_{T,D} = 20$, the relationship between the lease value and the tenant's debt value turns to be negative ($\Delta L/\Delta D = -11.68\%, -4.21\%$, respectively) confirming debt and lease as substitutes.

Agarwal et al. (2011) study the term structure of lease rates with endogenous default triggers for tenant capital structure assuming the landlord is default risk free. To compare the results, in Panel B we document the similar findings that leases and debt are complements and substitutes depending on tenant default risk. However, landlord default risk makes the findings more profound. The complementary behavior between debt and lease is also observed in the one-period analysis conducted by Lewis and Schallheim (1992). However, this effect is absent from the traditional literature that examines the term structure of lease rates. Thus, our analysis not only confirms that the tenant default risk is instrumental to observing this complementary behavior, but also confirms that the findings are valid when the counterparty risk exists.

6.4 The Term Structure of Leases

We now consider the lease term structure when tenant and landlord are subject to default risk. Figure 1 highlights the effects of changes in landlord and tenant debt maturities and riskiness on the equilibrium term structure of lease rates. The figure highlights the lease term structure that prevails under assumptions that both the landlord and tenant have short-term (5-year) debt and long-term (10-year) debt. In addition, we highlight the shift in the lease term structure that occurs when the landlord becomes risky. While the lease term structure is downward sloping, Figure 1 reveals two interesting results. First, when moving from a risk-free to a risky landlord the lease term structure becomes steeper, indicating that long-term leases are discounted in the presence of landlord risk. The intuition for the discount is that long-term leases increase tenant exposure to potential landlord default and, in equilibrium, the reduction in rent compensates the tenant for this increase in risk. Second, in the case of risky landlord and tenant, the impact of debt maturity dissipates as the lease term increases (the equilibrium rental rates converge). Notice that for risky landlord and tenant, the rental rate convergence begins approximately after the 10 year lease maturity; for the riskless landlord, convergence begins after the 15 year lease maturity. This phenomenon reflects how lengthening of lease maturities beyond both parties' debt maturity renders both short and medium term debt to be viewed as similar risks.

6.5 Impact of Tenant Default

Table 5 shows the relation between the probability of default on the tenant's existing debt and the lease term structure. As before, we examine the lease term structure when the tenant firm issues short-term debt (5-year), intermediate-term debt (10-year), and long-term debt (20-year). Italicized entries in each block indicate optimal endogenous default boundaries for tenant and landlord. These italicized entries along with the corresponding lease rate are the base case within each block. Within each block, we change the default boundary to highlight the impact of the probability of debt default. For a fixed optimal landlord boundary $(43.84, 43.75)$ and suboptimal tenant boundary $(30, 40, 60)$ we calculate the lease rate $r_{RR}^{T_L}$ satisfying equation (7) via the bisection method. Notice this task is much easier using suboptimal boundaries since the right-hand side of (7) is now independent of $r_{RR}^{T_L}$.

In Table 5, we first notice that the lease rate is increasing in the tenant default boundary. For example, when $T_L = 5$ and $T_{T,D} = 5$ the lease rate increases from 0.770 to 0.849 as the tenant default boundary increases. This is expected since the landlord should be compensated for increased tenant default likelihood.[24]

6.6 Impact of Taxes and Depreciation

Table 6 highlights how differences in landlord and tenant tax rates and changes in overall tax policy can affect the equilibrium lease rate. Recall from our model that the lease rate is a function of the landlord and tenant marginal corporate tax rates as well as the tax treatment of economic depreciation (q) as reflected in χ. As noted above, $\chi = 1$ reflects the case that accounting and economic depreciation are equivalent, while $\chi < 1$ reflects the condition that the tax deduction accepted with depreciation is less than that of the full economic depreciation. Thus, by varying χ, we can observe how changes in the depreciation schedules associated with the leased asset impact lease rates.

[24]Note that the lease rates in the rows corresponding to the default boundaries of 30, 40, and 60 of each block do not change (holding lease maturity constant) as tenant debt maturity increases ($T_{T,D} = 5, 10, 20$). This is due to the fact that once the tenant default boundary is determined, the tenant debt maturity does not enter into equation (7); the time variable in (7) refers to lease length T_L.

First, we consider how changes in the tenant's tax rate affect the lease rate. For a fixed landlord tax rate, the lease rate increases for higher tenant tax rates. For instance, when $\chi = 0.5$, and the landlord tax rate is 0.25, the lease rate increases from 0.782 to 0.819 as the tenant's tax rate increases from 0.25 to 0.40. This behavior, also observed in Agarwal et al. (2011), results from the incentives that higher tax rates create for the tenant to utilize more debt, which in turn, makes the firm riskier.

Similarly, we can observe how changes in the landlord's tax rate affect the lease rate. The results indicate that for a fixed tenant tax rate, the lease rate decreases for higher landlord tax rates; notice that the lease rates decreases from 0.782 to 0.748 as one moves along the first row of the table (holding tenant tax rate constant at 0.25). Once again, the incentive to utilize debt to take advantage of tax shields makes the landlord riskier for which the tenant is compensated in the form a lower lease rate.

However, changes in tax policies normally impact both firms simultaneously. The diagonal elements in each block in Table 6 show the impact on the equilibrium lease rate of increasing corporate taxes. Since the increase in corporate taxes alters both the landlord's and tenant's incentives to use debt in the same direction, Table 6 shows that the equilibrium lease rate remains virtually unchanged as tax rates increase. Thus, our analysis confirms that when both parties to a contract face the same tax environment, changes in tax policies should have no impact on the contract pricing. It is only in cases where changes in tax policy differentially impact one party over another that we should observe changes in the equilibrium contract pricing.

Finally, the effect of allowing the landlord to accelerate depreciation of the leased asset ($\chi = 0.5$ to $\chi = 1.5$) results in lower equilibrium lease rates; compare for example 0.782 (first row, first column) to 0.763 (seventh row, first column). This phenomenon is expected as tax benefits to the landlord are passed to the tenant in the form of lower equilibrium lease rates.

7 Empirical Analysis

In this section, we empirically validate several of the theoretical predictions from our model using a novel dataset of commercial real estate investments. In order to empirically test these predictions, we assembled a dataset of single-tenant commercial real estate properties utilizing data from the Trepp Data Feed loan file (Trepp) that comprises information on commercial real estate loans. Trepp provides data covering over 69,000 loans that underlie commercial mortgage-backed securities (CMBS). Trepp reports information about each loan and the property collateralizing the loan including information about the leases and square footage occupied by the property's largest tenants. Furthermore, each mortgage in the loan file has a series of bond payment dates, referred to as tape dates, that allow us to time stamp the information provided about the loan and property.

In addition to providing data about each loan on specific tape dates, Trepp provides information about the property and the property's tenants at the time the loan is securitized. However, information at the time of loan origination is somewhat limited. Thus, we use information about the loan either at the time it is securitized or from a tape date that occurs within 18 months of origination. Because a large number of loans tracked by Trepp are conduit loans (i.e. mortgages originated to be securitized), the characteristics of the loan at securitization should be a close proxy for the loan's characteristics at origination.

Although Trepp provides information on a large number of commercial mortgages, we apply a series of highly restrictive data screens in order to isolate properties where we can observe both the landlord and tenant capital structure. Since Trepp reports information about the top-three tenants in the property securing the mortgage, we use that information to create a dataset of single-tenant properties. For example, we identified 320 properties in the Trepp database as being "single-tenant" leased after screening all loans for properties that report the top tenant as occupying more than 80% of the total leaseable area with lease terms less than 40-years. We then hand screened each of the tenant names to identify 166 properties where the tenant was either a public company or a subsidiary of a public company at the time of lease origination. By screening for tenants that were public companies at lease origination, we are able to collect information about the tenant's capital structure (from publicly available financial statements.)

Trepp also provides the name of the borrower (property owner) for each mortgage. Thus, we examined the name of each property owner for the 166 properties with publicly traded tenants in order to determine whether the property owner was also a publicly traded firm.

Consistent with standard practice in commercial real estate, the Trepp borrower name field indicates that the mortgage borrowers were most likely single-entity limited liability corporations (LLC). As a result, we assume that each property is held by a single-asset firm allowing us to use the mortgage loan-to-value ratio as a proxy for the landlord's capital structure.

Panel A in Table 7 shows the distribution of properties by year of lease origination. We see that over 50% of the observations were originated in 2003 and 2004. Unfortunately, Trepp does not report the actual lease rate paid by the tenant. However, Trepp does report the property net operating income (NOI). Thus, under the assumption that tenants in single-tenant properties are usually responsible for expenses associated with the property, we use the property NOI as a proxy for the lease rent.[25] Panel B in Table 7 shows the descriptive statistics for the properties in our dataset. We see that landlords have an average capital structure (LTV) ratio of 66% while the tenant capital structure (debt/asset) ratio averages 59%. Furthermore, we see that 64% of the leases are classified as long-term (greater than 10-years.) In addition, we note that 62% of the properties are classified as "retail", 19% are industrial property, and 16% are general office buildings.

Our theoretical model predicts that the lease rate should be a function of both the landlord's capital structure (LTV) and the tenant's capital structure (debt/asset ratio). For example, Figure 1 shows that the impact of an increase in the landlord's capital structure (reflected by the movement from a riskfree landlord to a risky landlord) will have a greater impact on the lease rate as the lease maturity lengthens. In addition, keeping the endogenous default boundary of landlord fixed at its optimum, the theoretical predictions captured in Table 5 suggest that an increase in the tenant's debt-to-asset ratio (reflected in Table 5 by the increase in $\lambda_{D,T}$) will result in a decrease in the lease rate as the lease maturity increases. Furthermore, Table 5 indicates that as we hold debt and lease maturities constant (as reflected by the increase in the tenant default boundary (DB_T)), the lease rate will increase as the tenant's debt-to-asset ratio increases. On the other hand, in Table 2 within each block, we change the default boundary of the landlord and see the impact on tenant capital structure. We notice that an increase in landlord default probability will result in an

[25]Net operating income (NOI) is defined as gross revenues (rent) less operating expenses and is similar to earnings before interest, taxes, depreciation and amortization (EBITDA).

increase in the tenant's debt-to-asset ratio and a decrease in the lease rate.

We regress our proxy for the observed lease rate on measures of the landlord and tenant capital structure prior to the lease origination date. However, we also recognize that the landlord's capital structure (loan-to-value ratio) is endogenous to the associated mortgage terms and conditions prevailing in the capital markets. Thus, to account for the endogenous relation between loan terms (LTV, interest rate, and term) and NOI, we estimate the following system of equations:

$$
\begin{aligned}
LTV_i &= \alpha_0 + \alpha_1 r_i + \alpha_2 T_i + \alpha_3 NOI_i + \alpha_4 Debt/Asset_i + \epsilon_i \\
T_i &= \gamma_0 + \gamma_1 r_i + \gamma_2 LTV_i + \xi_i \\
r_i &= \delta_0 + \delta_1 LTV_i + \delta_2 T_i + \delta_3 r_f + \omega_i \\
NOI_i &= \beta_0 + \beta_1 LTV_i + \beta_2 (Debt/Asset_i) + \beta_3 (Long_i) \\
&\quad + \beta_4 (Long_i * LTV_i) + \beta_5 (Long_i * Debt/Asset_i) + \varepsilon_i
\end{aligned} \tag{20}
$$

where r_i is the mortgage interest rate at origination, T_i is the mortgage term for property i, r_f is the 10-year Constant Maturity Treasury rate; $Long_i$ is a dummy variable equal to one if the lease maturity is greater than 10-years, and zero otherwise, and $Debt/Asset_i$ is the tenant firm's debt-asset ratio at the quarter prior to lease origination.[26]

Table 8 reports the estimated coefficients. As predicted by our theoretical model, we see that the tenant capital structure has a negative and statistically significant estimated coefficient. The estimated coefficient indicate that a 1% increase in the tenant's capital structure (debt/asset) ratio decreases the lease rate by 0.94%. We also note that the estimated coefficient for the interaction of the dummy variable for leases that are longer than 10-years with the landlord's capital structure ($Long_i * LTV_i$) is negative and statistically significant. The negative coefficient on the interaction term indicates that increases in landlord debt usage have an even greater effect when leases are long-term than when leases are short-term. This result is exactly as predicted by the numerical analysis presented in Figure 1 showing an

[26] As noted above, NOI is our proxy for lease rent and is scaled by the leaseable area. We estimate the equations in (20) simultaneously using three-stage least squares instrumenting with the log of building age and property type. Our instruments satisfy the usual exclusion restrictions since CMBS mortgages are underwritten to uniform risk standards regardless of building age or type.

increasing negative relation between lease rates and lease term as landlord risk increases.

We also find a positive and statistically significant coefficient for the interaction of the tenant's capital structure and lease term ($Long_i * Debt/Asset_i$) indicating that long-term leases mitigate the impact of higher tenant debt use. Notice that the statistically significant negative $Debt/Asset_i$ coefficient (-0.944) and positive interaction coefficient (1.154) together imply that longer term leases should increase the lease amount at a faster rate than shorter maturity leases in response to an increase in the tenant's debt default risk (capital structure). This result is observed in Table 2: If we consider a change in tenant debt maturity from 5 years to 20 years and a lease maturity of 5 years, we find

$$\frac{\triangle r_{RR}}{\triangle\,(\text{Debt/Asset})} = \frac{0.7951 - 0.7871}{0.3686 - 0.3753} = -1.94.$$

Comparing this ratio to the change in tenant debt maturity from 5 years to 20 years and a lease maturity of 10 years producing

$$\frac{\triangle r_{RR}}{\triangle\,(\text{Debt/Asset})} = \frac{0.6684 - 0.6621}{0.1718 - 0.3810} = -0.03,$$

we see that the rate of change increased (-1.194 to -0.03). Thus, with long term leases, we find that a one unit increase in the debt-to-asset ratio of the tenant leads to a smaller decrease in the lease rate. The estimated coefficients for the interaction of long-term leases with tenant capital structure confirm this prediction.

8 Conclusion

The depth and length of the 2007-2009 financial crisis raised awareness of the implications of counterparty risk arising from capital structure decisions to many contracts once thought immune to such problems. Recent examples that demonstrate how a firm's capital structure can impact entities that have relationships with it extend well beyond the typical financial contracts discussed in the literature. For example, bankruptcies among franchisors, home builders, and real estate investors have highlighted that counterparty capital structure is an important risk to understand.

Using commercial real estate as the motivating example, we develop a continuous time structural model to consider how the endogenous capital structure decisions of landlords and tenants interact to determine equilibrium lease rates. Thus, we provide a novel mechanism to illustrate the credit contagion that results between tenant and landlord through the lease contract. Our analysis also highlights a little known aspect of how the riskiness of counterparties to a firm's off-balance sheet financing tools (such as leases) can impact the firm's capital structure decisions. As a result, our model illustrates the complexity and associated endogenous relationships that accompany corporate financing decisions.

Our numerical analysis provides a number of empirical predictions. First, our model predicts that tenants face lower equilibrium lease rates as their landlord's risk increases and this risk increases with exposure to the landlord through lease maturity. In addition, the numerical results show that credit contagion can be amplified through long-term off balance sheet contracts. In other words, when the landlord's credit condition deteriorates, tenant debt default probability increases through the interaction of the lease contract and the firm's capital structure. Second, our model indicates that tenants have an increasing preference for debt as their landlord riskiness increases. Third, our model confirms the intuitive phenomenon that landlords should be compensated with higher lease rates when renting to riskier firms. Fourth, our model provides the novel prediction that the downward sloping term structure of lease rates should become steeper as the landlord risk increases, indicating that long-term leases are discounted more heavily in the presence of landlord risk. Finally, we show that tenant default risk is instrumental in determining the complementary/substitution behavior between debt and leases. We observe debt and leases acting as complements for the tenant when their debt maturity is low regardless of landlord risk. However, this phenomenon changes to a substitutes effect as tenant debt maturity increases, once again, regardless of landlord risk. Thus, our numerical analysis offers novel insights into how debt and leases behave depending upon both individual and counterparty default risk.

In order to verify the veracity of the model, we empirically test the model's predictions using a dataset of single leased properties with publicly traded tenants. Our empirical analysis confirms the model's prediction that lease rates are negatively related to tenant and landlord capital structures. Furthermore, our analysis indicates that lease maturity has a differential impact on lease rates based on the lessor's risk, as predicted by the model.

A Derivations

A.1 Derivation of Lease Rate

Recall, δ is the market price of risk for the service value process. In this section, suppose τ is the time of default for the landlord and F is the cumulative distribution function for the landlord default time. We begin with four calculations which will assist in determining the lease rate. Using Fubini's theorem, we have

$$\begin{aligned}
\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{BD}(u)\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] &= \widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{BD}(0)e^{\left(\mu_S-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u+\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] \\
&= \int_0^t S_{BD}(0)e^{\left(\mu_S-r-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u} \times \widetilde{\mathbb{E}}\left[e^{\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\right]\mathrm{d}u \\
&= \int_0^t S_{BD}(0)e^{(\mu_S-r-\delta\sigma_S)u} \times (1-F(u;V_L,V_{L,B}))\mathrm{d}u,
\end{aligned}$$

where the last line follows by assuming the independence of τ and $\widetilde{W}_S(\cdot)$ since

$$\widetilde{\mathbb{E}}\left[e^{\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\right] = e^{\frac{u}{2}\sigma_S^2} \times (1-F(u;V_L,V_{L,B})).$$

Similarly,

$$\begin{aligned}
\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{AD}(u)\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] &= \widetilde{\mathbb{E}}\left[\int_0^t e^{-ru} S_{AD}(0)e^{\left(\mu_S-q-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u+\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right] \\
&= \int_0^t S_{AD}(0)e^{\left(\mu_S-r-q-\delta\sigma_S-\frac{\sigma_S^2}{2}\right)u} \times \widetilde{\mathbb{E}}\left[e^{\sigma_S\widetilde{W}_S(u)}\mathbb{1}_{\{\tau>u\}}\right]\mathrm{d}u \\
&= \int_0^t S_{AD}(0)e^{(\mu_S-r-q-\delta\sigma_S)u} \times (1-F(u;V_L,V_{L,B}))\mathrm{d}u
\end{aligned}$$

Let $n(\cdot)$ denote the density of the standard normal distribution. Using Fubini's theorem and the independence of τ and $\widetilde{W}_S(\cdot)$, we have

$$
\begin{aligned}
&\widetilde{\mathbb{E}}\left[\int_\tau^t e^{-rs} S_{BD}(s)\mathbb{1}_{\{\tau\le t\}}\mathrm{d}s\right] \\
&= S_{BD}(0)\int_{-\infty}^{\infty}\int_0^t\int_u^t e^{(\mu_S-r-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\mathrm{d}s\times f_L(u;V_L,V_{L,B})n(x)\ \mathrm{d}u\ \mathrm{d}x \\
&= S_{BD}(0)\int_0^t\int_u^t\int_{-\infty}^{\infty} e^{(\mu_S-r-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\times f_L(u;V_L,V_{L,B})\times\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\mathrm{d}x\ \mathrm{d}s\ \mathrm{d}u \\
&= S_{BD}(0)\int_0^t\int_u^t e^{(\mu_S-r-\delta\sigma_S)s}\mathrm{d}s\ f_L(u;V_L,V_{L,B})\mathrm{d}u \\
&= S_{BD}(0)\int_0^t \frac{1}{\mu_S-r-\delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}-e^{(\mu_S-r-\delta\sigma_S)u}\right)\ f_L(u;V_L,V_{L,B})\mathrm{d}u \\
&= \frac{S_{BD}(0)}{\mu_S-r-\delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-\delta\sigma_S)u}\ f_L(u;V_L,V_{L,B})\mathrm{d}u\right),
\end{aligned}
$$

if $\mu_S-r-\delta\sigma_S\neq 0$. Similarly, if $\mu_S-r-q-\delta\sigma_S\neq 0$,

$$
\begin{aligned}
&\widetilde{\mathbb{E}}\left[\int_\tau^t e^{-rs} S_{AD}(s)\mathbb{1}_{\{\tau\le t\}}\mathrm{d}s\right] \\
&= S_{AD}(0)\int_{-\infty}^{\infty}\int_0^t\int_u^t e^{(\mu_S-r-q-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\mathrm{d}s\times f_L(u;V_L,V_{L,B})n(x)\ \mathrm{d}u\ \mathrm{d}x \\
&= S_{AD}(0)\int_0^t\int_u^t\int_{-\infty}^{\infty} e^{(\mu_S-r-q-\delta\sigma_S-\frac{\sigma_S^2}{2})s+\sigma_S\sqrt{s}x}\times f_L(u;V_L,V_{L,B})\times\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\mathrm{d}x\ \mathrm{d}s\ \mathrm{d}u \\
&= S_{AD}(0)\int_0^t\int_u^t e^{(\mu_S-r-q-\delta\sigma_S)s}\mathrm{d}s\ f_L(u;V_L,V_{L,B})\mathrm{d}u \\
&= S_{AD}(0)\int_0^t \frac{1}{\mu_S-r-q-\delta\sigma_S}(e^{(\mu_S-r-q-\delta\sigma_S)t}-e^{(\mu_S-r-q-\delta\sigma_S)u})\times f_L(u;V_L,V_{L,B})\mathrm{d}u \\
&= \frac{S_{AD}(0)}{\mu_S-r-q-\delta\sigma_S}\left(e^{(\mu_S-r-q-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-q-\delta\sigma_S)u}\times f_L(u;V_L,V_{L,B})\mathrm{d}u\right)
\end{aligned}
$$

Recall from (3), the lessor's expected net cost of providing lease services is

$$
\begin{aligned}
&\widetilde{\mathbb{E}}\left[\int_0^t e^{-ru}[S_{BD}(u)-\chi_L Tax_L(S_{BD}(u)\mathrm{d}u-S_{AD}(u))]\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\right]+ \\
&+\widetilde{\mathbb{E}}\left[\rho_L^t\left(\int_\tau^t e^{-ru}[S_{BD}(u)-\chi_L Tax_L(S_{BD}(u)-S_{AD}(u))]\mathrm{d}u\right)\mathbb{1}_{\{\tau\le t\}}\right],
\end{aligned}
\tag{21}
$$

where the last term is the damage caused to the tenant, i.e., a proportion (perhaps greater than 1) of the future service flows. The four above calculations allow us to resolve this net cost of lease services. Thus, the first term in (21) is equal to

$$\begin{aligned}
&\widetilde{\mathbb{E}}\bigg[\int_0^t e^{-ru}[S_{BD}(u) - \chi_L Tax_L(S_{BD}(u)\mathrm{d}u - S_{AD}(u))]\mathbb{1}_{\{\tau>u\}}\mathrm{d}u\bigg] \\
&= (1-\chi_L Tax_L)\int_0^t S_{BD}(0)e^{(\mu_S - r - \delta\sigma_S)u} \times (1 - F(u;V_L,V_{L,B}))\mathrm{d}u + \\
&+ \chi_L Tax_L \int_0^t S_{AD}(0)e^{(\mu_S - r - q - \delta\sigma_S)u} \times (1 - F(u;V_L,V_{L,B}))\mathrm{d}u.
\end{aligned}$$

With regard to the second term in (21), we have

$$\begin{aligned}
&\widetilde{\mathbb{E}}\bigg[\rho_L^t\left(\int_\tau^t e^{-ru}[S_{BD}(u) - \chi_L Tax_L(S_{BD}(u) - S_{AD}(u))]\mathrm{d}u\right)\mathbb{1}_{\{\tau\le t\}}\bigg] \\
&= \rho_L^t\Bigg((1-\chi_L Tax_L)\times \\
&\times \frac{S_{BD}(0)}{\mu_S - r - \delta\sigma_S}\left(e^{(\mu_S - r - \delta\sigma_S)t}F(t;V_L,V_{L,B}) - \int_0^t e^{(\mu_S - r - \delta\sigma_S)u}\ f_L(u;V_L,V_{L,B})\mathrm{d}u\right) + \\
&+ \chi_L Tax_L \frac{S_{AD}(0)}{\mu_S - r - q - \delta\sigma_S}\times \\
&\times \left(e^{(\mu_S - r - q - \delta\sigma_S)t}F(t;V_L,V_{L,B}) - \int_0^t e^{(\mu_S - r - q - \delta\sigma_S)u} \times f_L(u;V_L,V_{L,B})\mathrm{d}u\right)\Bigg).
\end{aligned}$$

Now, equating (21) with $r_{RN}^t\left(\frac{1-e^{-rt}}{r}\right)$ yields,

$$
\begin{aligned}
& r_{RN}^t\left(\frac{1-e^{-rt}}{r}\right) = \\
& = (1-\chi_L Tax_L)\int_0^t S_{BD}(0)e^{(\mu_S-r-\delta\sigma_S)u}\times(1-F(u;V_L,V_{L,B}))\mathrm{d}u+ \\
& + \chi_L Tax_L\int_0^t S_{AD}(0)e^{(\mu_S-r-q-\delta\sigma_S)u}\times(1-F(u;V_L,V_{L,B}))\mathrm{d}u + \\
& + \rho_L^t\Bigg((1-\chi_L Tax_L)\times \\
& \times\frac{S_{BD}(0)}{\mu_S-r-\delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-\delta\sigma_S)u}\ f_L(u;V_L,V_{L,B})\mathrm{d}u\right)+ \\
& + \chi_L Tax_L\frac{S_{AD}(0)}{\mu_S-r-q-\delta\sigma_S}\times \\
& \times\left(e^{(\mu_S-r-q-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-q-\delta\sigma_S)u}\times f_L(u;V_L,V_{L,B})\mathrm{d}u\right)\Bigg).
\end{aligned}
\tag{22}
$$

Solving the above equation for r_{RN}^t yields the risky lessor, risk-free tenant lease rate:

(23)

$$
\begin{aligned}
r_{RN}^t = & \frac{r}{1-e^{-rt}}\times \\
& \times\Bigg[(1-\chi_L Tax_L)\int_0^t S_{BD}(0)e^{(\mu_S-r-\delta\sigma_S)u}\times(1-F(u;V_L,V_{L,B}))\mathrm{d}u+ \\
& + \chi_L Tax_L\int_0^t S_{AD}(0)e^{(\mu_S-r-q-\delta\sigma_S)u}\times(1-F(u;V_L,V_{L,B}))\mathrm{d}u + \\
& + \rho_L^t\Bigg((1-\chi_L Tax_L)\times \\
& \times\frac{S_{BD}(0)}{\mu_S-r-\delta\sigma_S}\left(e^{(\mu_S-r-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-\delta\sigma_S)u}\ f_L(u;V_L,V_{L,B})\mathrm{d}u\right)+ \\
& + \chi_L Tax_L\frac{S_{AD}(0)}{\mu_S-r-q-\delta\sigma_S}\times \\
& \times\left(e^{(\mu_S-r-q-\delta\sigma_S)t}F(t;V_L,V_{L,B})-\int_0^t e^{(\mu_S-r-q-\delta\sigma_S)u}\times f_L(u;V_L,V_{L,B})\mathrm{d}u\right)\Bigg)\Bigg].
\end{aligned}
$$

A.2 Definitions of Constants

$$a_T := \frac{r - \delta_{V_T} - (\sigma^2_{V_T}/2)}{\sigma^2_{V_T}};\ a_L := \frac{r - \delta_{V_L} - (\sigma^2_{V_L}/2)}{\sigma^2_{V_L}};$$

$$b_T := \ln\left(\frac{V_T}{V_{T,B}}\right);\ b_L := \ln\left(\frac{V_L}{V_{L,B}}\right);$$

$$z_T := \frac{((a_T\sigma^2_{V_T})^2 + 2r\sigma^2_{V_T})^{1/2}}{\sigma^2_{V_T}};\ z_L := \frac{((a_T\sigma^2_{V_L})^2 + 2r\sigma^2_{V_L})^{1/2}}{\sigma^2_{V_L}};$$

$$x_T := a_T + z_T; x_L := a_L + z_L;$$

$$\begin{aligned} A := {} & 2a_L e^{-rT_{L,D}} N(a_L\sigma_{V_L}\sqrt{T_{L,D}}) - 2z_L N(z_L\sigma_{V_L}\sqrt{T_{L,D}}) \\ & - \frac{2}{\sigma_{V_L}\sqrt{T_{L,D}}} n(z_L\sigma_{V_L}\sqrt{T_{L,D}}) + \frac{2e^{-rT_{L,D}}}{\sigma_{V_L}\sqrt{T_{L,D}}} n(a_L\sigma_{V_L}\sqrt{T_{L,D}}) + (z_L - a_L); \end{aligned}$$

$$B := -\left(2z_L + \frac{2}{z_L\sigma^2_{V_L}T_{L,D}}\right) N(z_L\sigma_{V_L}\sqrt{T_{L,D}}) - \frac{2}{\sigma_{V_L}\sqrt{T_{L,D}}} n(z_L\sigma_S\sqrt{T_{L,D}}) + (z_L - a_L) + \frac{1}{z_L\sigma^2_{V_L}T_{L,D}};$$

$$K_1^T := A/(rT)$$

$$K_2^T := B$$

$$K_3 := (C_T + \Omega_R)\left(\frac{Tax_T}{r}\right) x_T;$$

$$K_4 := (1 - \rho_R)\Omega_R \left(\frac{1 - e^{-r\frac{T_L}{2}}}{r}\right);$$

$$M := \left(\frac{\Omega_R}{r}\rho_R\right)\left(K_1^{T_L}\frac{T_L}{T_{T,D}} - K_2^{T_{T,D}}\right) - \left(\frac{\Omega_R}{r}\frac{\rho_R}{T_L}\right)(K_1^{T_L} - K_2^{T_L}),$$

where $N(\cdot)$ is the cumulative standard normal distribution, and $n(\cdot)$ is the probability density function of the standard normal distribution. Tax_T is tenant's corporate tax rate.

A.3 Calculation for "damage by default" term in (17)

The "damage by default" term is the compensation owed to the tenant upon landlord default and is equal to

$$\rho_L \widetilde{\mathbb{E}}\left[\left(\int_{\tau_L}^{T_L} e^{-r(u-\frac{T_L}{2})}[S_{BD}(u) - \chi_L Tax_L(S_{BD}(u) - S_{AD}(u))]\mathrm{d}u\right)\mathbb{1}_{\{\tau_L \le T_L\}}\right]$$

where ρ_L is the recovery rate for lost service flows and δ is the market price of risk parameter. Note that this cost depends upon the endogenous default time for the landlord. To simplify the analysis, we calculate this cost when default occurs at the midpoint of the lease contract. This simplification does not assume that the landlord's endogenous default time τ_L is fixed at $T_L/2$. Rather, we are simplifying the cost calculation for the landlord. The choice of using the midpoint of the lease contract serves as a means of calculating the average cost incurred by the landlord. Thus, we have

$$\begin{aligned}
\text{damage by default} &= \rho_L \widetilde{\mathbb{E}}\left[\int_{T_L/2}^{T_L} e^{-r(u-\frac{T_L}{2})}[S_{BD}(u) - \chi_L Tax_L(S_{BD}(u)\mathrm{d}u - S_{AD}(u))]\mathrm{d}u\right] \\
&= \rho_L e^{r\frac{T_L}{2}}\left[(1-\chi_L Tax_L)\frac{S_{BD}(0)}{\mu_S - r - \delta\sigma_S}\left(e^{(\mu_S - r - \delta\sigma_S)T_L} - e^{(\mu_S - r - \delta\sigma_S)\frac{T_L}{2}}\right)\right. \\
&\quad \left. + \frac{S_{AD}(0)}{\mu_S - r - \delta\sigma_S - q}\chi_L Tax_L\left(e^{(\mu_S - r - \delta\sigma_S - q)T_L} - e^{(\mu_S - r - \delta\sigma_S - q)\frac{T_L}{2}}\right)\right], \\
&\quad \text{if } \mu_S - \delta\sigma_S - q - r \neq 0.
\end{aligned}$$

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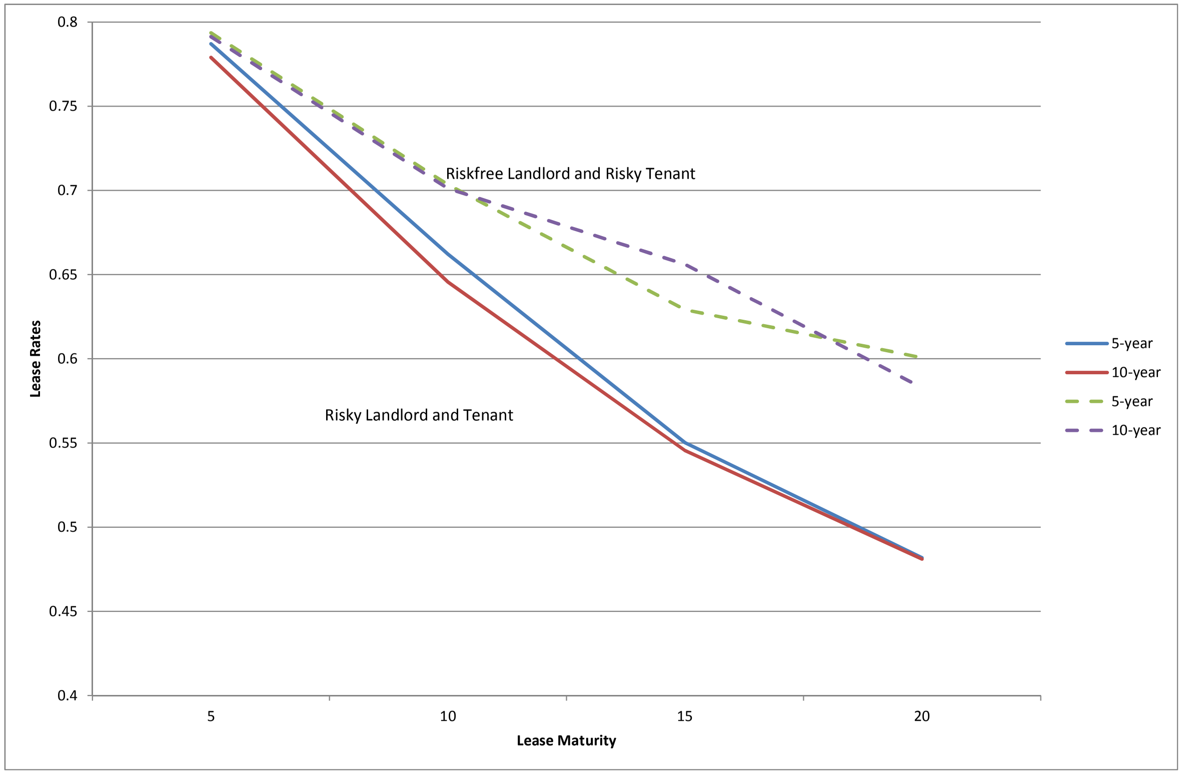

Figure 1: Term Structure of Lease Rates

Notes: Figure 1 highlights the term structure of lease rates using the model parameters from our base case.

Parameters: $r = 0.075$, landlord corporate tax rate $Tax_L = 0.35$; tenant corporate tax rate $Tax_T = 0.35$;$\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5, 10$; debt maturity for tenant $T_{T,D} = 5, 10, 20$; debt maturity for landlord $T_{L,D} = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$.

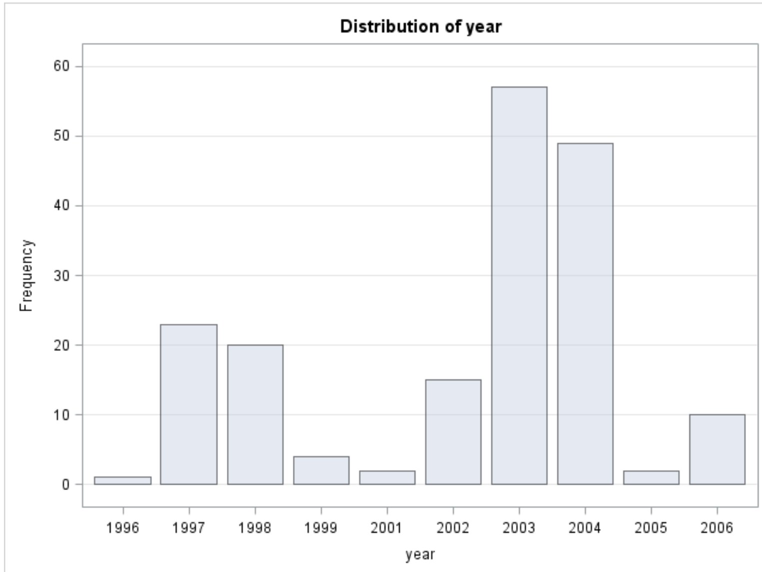

Figure 2: Property Distribution

Notes: Figure 2 highlights the property distributions by year

Table 1: Initial Parameter Values

Base Case Parameters			
Market Parameters		Landlord Parameters	
r	0.075	μ_{V_L}	0.05
Tax_L, Tax_T	0.35	σ_{V_L}	0.06
μ_S	0.06	$V_L(0)$	100
σ_S	0.2	Cost of bankruptcy	0.5
q	0.05	Default Recovery	0.62
S_0	1		
market price of risk δ	0.83	Tenant Parameters	
		μ_{V_T}	0.05
		σ_{V_T}	0.2
		δ_{V_T}	0.06
		$V_T(0)$	100
		Cost of bankruptcy	0.5
		Default Recovery	0.62

Notes: Base case parameters match those in the literature allowing for comparison of our results with previous studies; see for example, Agarwal et al. (2011).

Table 2: The Impact of Landlord Default Probability on Lease Rate Term Structure

Landlord Default		Tenant Default		Lease Rates			Tenant Capital Structure			Landlord Capital Structure	
Boundary	Probability	Boundary	Probability	r_{RN}	r_{RR}	P	C	Value	Debt/Value	Value	Debt/Value
Lease Maturity = 5-years, Tenant Debt Maturity = 5-years											
20	0.0010	46.83	0.1499	0.7754	0.7935	46.50	4.19	126.4834	0.3749	193.05	0.3663
30	0.0167	46.63	0.1499	0.7754	0.7935	46.50	4.19	126.4837	0.3749	186.05	0.3929
43.84	*0.1163*	*46.57*	*0.1492*	*0.7693*	*0.7871*	*46.50*	*4.19*	*126.3739*	*0.3753*	*110.96*	*0.4769*
60	0.3600	46.47	0.1480	0.7121	0.7284	47.00	4.23	125.2764	0.3827	147.58	0.5439
Lease Maturity = 5-years, Tenant Debt Maturity = 10-years											
20	0.0010	45.67	0.3625	0.7754	0.7913	44.00	3.96	126.1787	0.3467	193.05	0.3663
30	0.0167	45.67	0.3625	0.7754	0.7913	44.00	3.96	126.1789	0.3467	186.05	0.3929
43.84	*0.1163*	*45.56*	*0.3607*	*0.7693*	*0.7849*	*44.00*	*3.96*	*126.0976*	*0.3472*	*110.96*	*0.4769*
60	0.3600	45.22	0.3553	0.7121	0.7258	45.00	4.05	125.2830	0.3587	147.58	0.5439
Lease Maturity = 5-years, Tenant Debt Maturity = 20-years											
20	0.0010	49.63	0.6639	0.7754	0.8019	51.00	4.59	126.2204	0.3676	193.05	0.3663
30	0.0167	49.63	0.6639	0.7754	0.8019	51.00	4.59	126.2205	0.3676	186.05	0.3929
43.84	*0.1163*	*49.48*	*0.6622*	*0.7693*	*0.7951*	*51.00*	*4.59*	*126.1682*	*0.3686*	*110.96*	*0.4769*
60	0.3600	48.40	0.6495	0.7121	0.7330	51.50	4.64	125.6277	0.3803	147.58	0.5439
Lease Maturity = 10-years, Tenant Debt Maturity = 5-years											
20	0.0010	48.28	0.1720	0.6387	0.7031	49.00	4.90	140.2134	0.3735	191.92	0.3703
30	0.0167	48.23	0.1713	0.6339	0.6976	49.00	4.90	140.0145	0.3739	184.98	0.3969
43.75	*0.1151*	*48.35*	*0.1728*	*0.6011*	*0.6621*	*49.50*	*4.95*	*138.5962*	*0.3810*	*110.95*	*0.4769*
60	0.3600	49.25	0.1855	0.5151	0.5715	51.50	5.15	134.7787	0.4060	146.79	0.5484
Lease Maturity = 10-years, Tenant Debt Maturity = 10-years											
20	0.0010	47.86	0.3975	0.6387	0.7008	31.50	3.15	134.3939	0.2452	191.92	0.3703
30	0.0167	47.63	0.3939	0.6339	0.6944	31.50	3.15	134.2619	0.2457	184.98	0.3969
43.75	*0.1151*	*47.87*	*0.3977*	*0.6011*	*0.6597*	*33.50*	*3.35*	*133.3225*	*0.2628*	*110.95*	*0.4769*
60	0.3600	48.26	0.4040	0.5151	0.5670	38.50	3.85	130.8456	0.3069	146.79	0.5484
Lease Maturity = 10-years, Tenant Debt Maturity = 20-years											
20	0.0010	50.70	0.6762	0.6387	0.7174	24.50	2.21	130.1120	0.1503	191.92	0.3703
30	0.0167	50.77	0.6769	0.6339	0.7125	25.00	2.25	130.0106	0.1541	184.98	0.3969
43.75	*0.1151*	*49.53*	*0.6627*	*0.6011*	*0.6684*	*26.50*	*2.39*	*129.3810*	*0.1718*	*110.95*	*0.4769*
60	0.3600	54.39	0.7160	0.5151	0.5997	38.00	3.80	127.7404	0.2624	146.79	0.5484

Notes: Table 2 examines the relationship between the lease term structure and the probability of default on the landlord's debt.

Parameters: $r = 0.075$, landlord corporate tax rate $Tax_L = 0.35$; tenant corporate tax rate $Tax_T = 0.35$; $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5, 10$; debt maturity for tenant $T_{T,D} = 5, 10, 20$; debt maturity for landlord $T_{L,D} = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$. Principal and Coupon Window used in the implementation: $P = [0.5, 100]$ using 0.5 as the principal step size, $C = [(0.01)P, (0.1)P]$ with 0.01 as the coupon step size.

Description: The first and second columns are the endogenous landlord bankruptcy boundary and the landlord's default on debt probability (not scaled by 100), i.e.,

$$\mathbb{P}_{V_L(0)=100}[V_L(\tau_L) \leq T_{L,D}].$$

The third and fourth columns are the endogenous tenant bankruptcy boundary and the tenant's default on debt probability (not scaled by 100), i.e.,

$$\mathbb{P}_{V_T(0)=100}[V_T(\tau_T) \leq T_{T,D}].$$

The fifth and sixth columns are the risky landlord, risk-free tenant lease rates. The seventh through tenth columns make up the tenant's optimal capital structure. Specifically, the seventh and eight columns are the optimal Principal and Coupon for the tenant firm. The ninth column is the optimal tenant firm value and the tenth column is the optimal leverage ratio for the tenant firm. The eleven through twelve columns make up the landlord's optimal capital structure.

Note that, in each block, the italicized third row indicates the optimal capital structure for the landlord and tenant firms when $T_{L,D} = 5$.

Table 3: The Impact of Short-term and Long-term Leases

	Landlord Default		Tenant Default		Leases	
	Boundary	Probability	Boundary	Probability	r_{RN}	r_{RR}
Lease Maturity = 5-years, Tenant Debt Maturity = 5-years						
Landlord Debt Maturity = 5-years	43.84	0.1163	46.57	0.1492	0.7693	0.7871
Landlord Debt Maturity = 10-years	47.17	0.3866	46.52	0.1485	0.7638	0.7814
Lease Maturity = 5-years, Tenant Debt Maturity = 10-years						
Landlord Debt Maturity = 5-years	43.84	0.1163	45.56	0.3607	0.7693	0.7849
Landlord Debt Maturity = 10-years	47.17	0.3866	45.45	0.3590	0.7638	0.7790
Lease Maturity = 5-years, Tenant Debt Maturity = 20-years						
Landlord Debt Maturity = 5-years	43.84	0.1163	49.48	0.6622	0.7693	0.7951
Landlord Debt Maturity = 10-years	47.17	0.3866	49.34	0.6606	0.7638	0.7890
Lease Maturity = 10-years, Tenant Debt Maturity = 5-years						
Landlord Debt Maturity = 5-years	43.75	0.1151	48.35	0.1728	0.6011	0.6621
Landlord Debt Maturity = 10-years	47.05	0.3846	48.65	0.1771	0.5872	0.6484
Lease Maturity = 10-years, Tenant Debt Maturity = 10-years						
Landlord Debt Maturity = 5-years	43.75	0.1151	47.87	0.3977	0.6011	0.6597
Landlord Debt Maturity = 10-years	47.05	0.3846	48.11	0.4015	0.5872	0.6456
Lease Maturity = 10-years, Tenant Debt Maturity = 20-years						
Landlord Debt Maturity = 5-years	43.75	0.1151	49.53	0.6627	0.6011	0.6684
Landlord Debt Maturity = 10-years	47.05	0.3846	49.33	0.6605	0.5872	0.6519

Table 4: Debt and Lease as Complements/Substitutes

Panel A: Landlord is default risky; Tenant is default risky

	$V^*_{L,B}$	$\lambda_{D,L}$	$V^*_{T,B}$	V^*_T	$\lambda_{D,T}$	$(D/V)^*_T$	r_{RR}	r_{RN}	Lease Value	Tenant′s Debt Value	$\Delta L/\Delta D$
$T_{T,D}$=5											
T_L=5	43.84	0.1163	46.57	126.37	0.149	37.53%	0.7871	0.7693	3.2076	47.4267	
T_L=10	43.75	0.1151	48.35	138.60	0.173	38.10%	0.6621	0.6011	4.2288	52.7914	18.99%
$T_{T,D}$=10											
T_L=5	43.84	0.1163	45.56	126.09	0.361	34.72%	0.7849	0.7693	3.2076	43.7784	
T_L=10	43.75	0.1151	47.87	133.32	0.398	26.28%	0.6597	0.6011	4.2288	35.0365	−11.68%
$T_{T,D}$=20											
T_L=5	43.84	0.1163	49.48	126.16	0.662	36.86%	0.7951	0.7693	3.2076	51.0822	
T_L=10	43.75	0.1151	49.53	129.38	0.663	17.18%	0.6684	0.6011	4.2288	32.4485	−4.21%

Panel B: Landlord is default risk free; Tenant is default risky

	$V^*_{L,B}$	$\lambda_{D,L}$	$V^*_{T,B}$	V^*_T	$\lambda_{D,T}$	$(D/V)^*_T$	r_{NR}	r_{NN}	Lease Value	Tenant′s Debt Value	$\Delta L/\Delta D$
$T_{T,D}$=5											
T_L=5	0.0000	0.0000	46.6254	126.4833	0.1499	37.49%	0.7935	0.7754	3.2330	47.4233	
T_L=10	0.0000	0.0000	48.2836	140.2215	0.1720	37.35%	0.7034	0.6389	4.4945	52.3744	25.48%
$T_{T,D}$=10											
T_L=5	0.0000	0.0000	45.6713	126.1786	0.3625	34.67%	0.7913	0.7754	3.2330	43.7480	
T_L=10	0.0000	0.0000	47.8664	134.3993	0.3977	24.52%	0.7011	0.6389	4.4945	32.9559	−11.69%
$T_{T,D} = 20$											
T_L=5	0.0000	0.0000	49.6297	126.2203	0.6639	36.76%	0.8019	0.7754	3.2330	46.3983	
T_L=10	0.0000	0.0000	50.7192	130.1154	0.6763	15.02%	0.7177	0.6389	4.4945	19.5455	−4.70%

Parameters: $r = 0.075$, landlord corporate tax rate $Tax_L = 0.35$; tenant corporate tax rate $Tax_T = 0.35$; $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5, 10$; debt maturity for tenant $T_{T,D} = 5, 10, 20$; debt maturity for landlord $T_{L,D} = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$.

Description: The Panel A is the result for default risky landlord and default risky tenant. The Panel B is the result for default risk-free landlord and defautl risky tenant by assuming landlord's optimal bankrutcy boundary is 0. In each panel, the first column is the endogenous landlord bankruptcy boundary ($V^*_{L,B}$). The second column is the landlord bankruptcy probability of debt ($\lambda_{D,L}$).The third column is the tenant bankruptcy boundary ($V^*_{T,B}$). The fourth column tenant's optimal total firm value (V^*_T). The fifth column is the tenant bankruptcy probability of debt ($\lambda_{D,T}$). The sixth column is tenant's optimal leverage ratio($(D/V)^*_T$) . The seventh column is the lease rate when both landlord and tenant are default risky (r_{RR}). The eighth column is the lease rate only landlord is default risky(r_{RN}). The nighth column is the lease contract value. The tenth column is tenant's optimal debt value. The eleventh column is the ratio of tenant's lease value change over debt value change ($\Delta L/\Delta D$) as lease maturity increases from 5 years to 10 years.

Table 5: The Impact of Tenant Default Boundary on Lease Rate Term Structure

$T_L = 5$					$T_L = 10$				
$V^*_{L,B}$	$V^*_{T,B}$	$\lambda_{D,T}$	$\lambda_{D,L}$	r_{RR}	$V^*_{L,B}$	$V^*_{T,B}$	$\lambda_{D,T}$	$\lambda_{D,L}$	r_{RR}
$T_{T,D} = 5$					$T_{T,D} = 5$				
	30			0.7702		30			0.6086
	40			0.7760		40			0.6286
43.84	*46.57*	0.1492	0.1163	0.7871	*43.75*	*48.35*	0.1728	0.1151	0.6621
	60			0.8487		60			0.7491
$T_{T,D} = 10$					$T_{T,D} = 10$				
	30			0.7702		30			0.6086
	40			0.7760		40			0.6286
43.84	*45.56*	0.3607	0.1163	0.7849	*43.75*	*47.87*	0.3977	0.1151	0.6597
	60			0.8487		60			0.7491
$T_{T,D} = 20$					$T_{T,D} = 20$				
	30			0.7702		30			0.6086
	40			0.7760		40			0.6286
43.84	*49.48*	0.6622	0.1163	0.7951	*43.75*	*49.53*	0.6627	0.1151	0.6684
	60			0.8487		60			0.7491

Parameters: $r = 0.075$, landlord corporate tax rate $Tax_L = 0.35$; tenant corporate tax rate $Tax_T = 0.35$; $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5, 10$; debt maturity for tenant $T_{T,D} = 5, 10, 20$; debt maturity for landlord $T_{L,D} = 5$, lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$.

Description: The first column is the endogenous landlord bankruptcy boundary ($V^*_{L,B}$). The second column is the tenant bankruptcy boundary ($V^*_{T,B}$). The third row in each block (where block refers to entries corresponding to a $(T_L, T_{T,D})$ pair, $T_{T,D}$ is the maturity of tenant's debt) is the optimal endogenous boundary found using the landlord's endogenous boundary in the first column. The third and fourth columns are the default probabilities of debt for both tenant ($\lambda_{D,T}$) and landlord ($\lambda_{D,L}$) respectively. The fifth column is the lease rate implied using the first two columns. Columns six through ten are repeats of columns one-three using $T_L = 10$. Italicized table entries indicate optimal tenant and landlord boundary values.

Table 6: The Impact of Taxes and Depreciation on Lease Rate Term Structure

Tenant Tax Rate	Landlord Tax Rate			
$\chi = 0.5$	0	0.25	0.35	0.40
0	0.7890	0.7792	0.7963	0.7450
0.25	0.7920	0.7821	0.7721	0.7478
0.35	0.8067	0.7974	0.7871	0.7618
0.40	0.8294	0.8187	0.8080	0.7817
$\chi = 1$				
0	0.7890	0.7695	0.7567	0.7370
0.25	0.7920	0.7723	0.7597	0.7397
0.35	0.8067	0.7873	0.7740	0.7534
0.40	0.8294	0.8082	0.7944	0.7748
$\chi = 1.5$				
0	0.7890	0.7598	0.7448	0.7291
0.25	0.7920	0.7628	0.7476	0.7317
0.35	0.8067	0.7772	0.7615	0.7461
0.40	0.8294	0.7977	0.7814	0.7662

Notes: Table 6 highlights how differences in landlord and tenant tax rates as well changes in overall tax policy affect the lease rate.

Parameters: $r = 0.075$, $\mu_S = 0.06$; $\sigma_S = 0.2$; $q = 0.05$; initial service flow $S_0 = 1$; market price of risk $\delta = 0.83$, level that accounting depreciation is scaled to economic depreciation $\chi = 0.5$. Landlord Parameters: $\mu_{V_L} = 0.05$; $\sigma_{V_L} = 0.2$; $\delta_{V_L} = 0.06$; $V_L(0) = 100$. Tenant Parameters: $\mu_{V_T} = 0.05$; $\sigma_{V_T} = 0.2$; $\delta_{V_T} = 0.06$; $V_T(0) = 100$. Dates: Lease maturity $T_L = 5$; debt maturity for tenant $T_{T,D} = 5$; debt maturity for landlord $T_{L,D} = 5$; lifetime of leased asset $T_{\text{Life}} = 30$. Costs and Recovery values: Cost of bankruptcy for tenant $\alpha_T = 0.5$; cost of bankruptcy for landlord $\alpha_L = 0.5$; recovery to tenant for landlord default $\rho_L = 0.62$; recovery to landlord for tenant default $\rho_R = 0.62$; aggregate recovery to holder's of tenant debt $\rho_{T,D} = 1$; aggregate recovery to holder's of landlord debt $\rho_{L,D} = 1$.

Description: The assumed tax rates $0.25, 0.35, 0.4$ for the tenant appear as rows and the assumed tax rates for the landlord $0.25, 0.35, 0.4$ appear as columns. Entries of the table are the lease rate for a risky landlord and tenant.

Table 7: Summary Characteristics of the Single-Tenant Property Sample

Panel A: Distribution of properties by year of lease origination		
Year	**Frequency**	**Percent**
1996	1	0.6
1997	21	12.7
1998	17	10.2
1999	4	2.4
2001	2	1.2
2002	13	7.8
2003	47	28.3
2004	49	29.5
2005	2	1.2
2006	10	6.0
Total	166	100.0

Panel B: Descriptive statistics		
Landlord:	**Mean**	**Std. Dev.**
Loan-to-Value (LTV_i)	65.925	16.677
Interest Rate Spread (r_i)	1.730	0.664
Mortgage Term (T_i)	144.584	54.465
Tenant:		
Lease Rate (NOI_i/sf)	29.320	48.797
Lease Term (Months)	154.030	81.093
Long-term Lease Indicator ($Long_i$)	0.639	0.482
Debt/Asset Ratio at Lease Origination ($Debt/Asset_i$)	58.991	22.392
Property Characteristics:		
Building Age	13.777	19.709
Industrial Property Indicator	0.187	0.391
Office Property Indicator	0.157	0.365
Retail Property Indicator	0.620	0.487
Other Property Type	0.036	0.187

Notes: Table 7 reports the descriptive statistics for the commercial real estate properties identified from commercial real estate loans that are contained in the Trepp Data Feed file (Trepp). Trepp provides information on loans that underlie commercial mortgage-backed securities (CMBS). The data comprises property-level and loan-level characteristics including the loan-to-value (LTV) at origination, the loan contract interest rate less the 10-year constant maturity treasury rate (r_i), the mortgage term (T), and the property net operating income (NOI). Our dataset comprises loans on "single-tenant" properties where the tenant is identified as either a public company or a subsidiary of a public company at the time of lease origination. By screening for tenants that were public companies at lease origination, we are able to collect information about the tenant's capital structure (*Debt_Asset* ratio) from publicly available financial statements.

Table 8: Three-stage Lease Squares (3SLS) Estimation of Landlord and Tenant Capital Structure on Lease Rate

	LTV	Mortgage Term	Interest Rate	NOI Rate
Intercept	84.439***	331.528***	3.303***	224.520***
Interest Rate (r)	−1.365	3.674*		
Mortgage Term (T)	−0.044***		−0.004	
NOI Rate	−0.137***			
Tenant ($Debt/Asset$)	0.014			−0.944***
Landlord (LTV)		−3.180	−0.002	−1.553*
10-yr Treasury Rate			0.814***	
Lease Term ($Long$)				224.911***
$LTV * LONG$				−5.070***
$Debt/Asset * Long$				1.154***
System Weighted MSE	8.592			
Degrees of freedom	638			
System Weighted Rsq	0.647			

Notes: Table 8 reports the estimated coefficients for the following system of equations:

$$
\begin{aligned}
LTV_i &= \alpha_0 + \alpha_1 r_i + \alpha_2 T_i + \alpha_3 NOI_i + \alpha_4 Debt/Asset_i + \epsilon_i \\
T_i &= \gamma_0 + \gamma_1 r_i + \gamma_2 LTV_i + \xi_i \\
r_i &= \delta_0 + \delta_1 LTV_i + \delta_2 T_i + \delta_3 r_f + \omega_i \\
NOI_i &= \beta_0 + \beta_1 LTV_i + \beta_2 (Debt/Asset_i) + \beta_3 (Long_i) \\
&\quad + \beta_4 (Long_i * LTV_i) + \beta_5 (Long_i * Debt/Asset_i) + \varepsilon_i
\end{aligned}
\tag{24}
$$

where LTV_i, r_i, r_T, and T_i are the mortgage loan-to-value ratio, contract interest rate, the 10-year constant maturity treasury, and mortgage term for property i, respectively; $Long_i$ is a dummy variable equal to one if the lease maturity is greater than 10-years, and zero otherwise; $Debt/Asset_i$ is the tenant's debt to asset ratio at time of lease origination; and NOI_i is a proxy for lease rent and is scaled by the leaseable area. The system is estimated via three-stage lease squares (3SLS).